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THE TENSORPOLYNOMIAL FAILURE CRITERION FOR WOOD

DERIVATION AND EXPLANATION OF:

the failure criterion of wood and the meaning of the polynomial constants;
the critical distortional energy principle of orthotropic composites like wood;
the generalized and extended Hankinson equations; the Norris- Tsai Wu- Hoffmann- Coulomb- and Wu- fracture criteria.

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CONTENT

1. INTRODUCTION

The now used failure criteria for wood are not generally valid and consistent and thus should not be used. The general valid approach was derived long ago [1] and is as easy to apply as the now used approximations. The behaviour at "flow" and failure is given in [6] in the main directions (see e.g. in general fig. 5.6 of [6]). From this behaviour, the failure criterion could be derived. This however will give a complicated formula of the failure surface and a simplification then is possible by a polynomial expansion of that failure surface. This expansion of the failure surface in stress space into a polynomial, consisting of a linear combination of orthogonal polynomials, provides easily found constants (by the orthogonality property) when the expanded function is known, and the row can be extended, when necessary for a higher precision, without changing the already determined constants of the row. When choosing in advance a limited number of terms of the polynomial, up to some degree, the expansion procedure need not to be performed, because the result is in principle identical to a least square fit of the data to a polynomial of that chosen degree. This choice of the number of terms may depend on the wanted precision of the expansion and on the practical use. The choice of the number of higher order terms does not influence the values of the second and first order terms and the quadratic polynomial part has a special meaning. The third order terms represent special hardening effects and only a few of such terms are necessary for a precise description of the strength of wood in longitudinal direction. In the transverse direction, no higher order terms are needed and only the second order polynomial is enough for a precise description [1] because hardening leads to a less orthotropic (quasi isotropic) behaviour. The special property of the quadratic form of the polynomial was discussed in [1] where it was shown to represent the critical distortional energy principle of yield. The mentioned simple mathematical steps of the derivation were left out of the paper and are given here. Based on this principle of a polynomial expansion of the real failure surface, a general failure criterion, satisfying equilibrium in all directions, was for wood first derived in [1] and the most important aspects can be found in that publication. The in [1] given explanation of the existing criteria and the approximation of the coupling terms like F_{12}, are verified, e.g. in Madison [2], where it was shown that Cowins approximation [12] does not apply for wood. As extension, the derivation is given here (and in [7]) of an exact modified Hankinson criterion and of the general form of the higher order constants and how the constants of the polynomial can be determined from uniaxial off-axis tests in the main planes.
An extension of the method was given [3] by a general approach for anisotropic, not orthotropic, behaviour of joints, (as punched out metal plates) and the simplification of the transformations by 2 angles as variables.
A confirmation of the results of [1] by means of coherent measurements (only in the radial-longitudinal plane) and the generalization to an equivalent, quasi homogeneous, failure criterion for wood with small defects is given in [4]. These

measurements also show a determining influence of hardening and oriented micro-crack propagation on the equivalent main strengths and the failure criterion of wood. This follows from the theoretical explanation in [9] of the Wu fracture mechanics criterion for layered composites that is based on oriented (in grain direction) crack propagation of flat elliptic micro cracks in the matrix. It here is shown that this Wu- (or Mohr-) criterion is also determining the failure criterion of wood, showing the same oriented micro cracks to be responsible for failure in general.
The positive third order terms of the polynomial are shown to represent this hardening effect due to micro crack arrest by strong layers.
It is shown that the, usually applied, von Mises- Hill- Hoffmann- Hankinson- and Norris criteria are special forms of the critical distortional energy principle of yield and are not generally valid. The Hill- and Norris- criteria only apply for materials with equal compression and tension strengths. Only the Hoffmann criterion accounts for different strengths. However the Hill- and Hoffmann criteria contain a cyclic symmetry of the stresses in the quadratic terms, as applies for the isotropic case what causes a fixed, not free, orientation of the failure ellipse in stress space [1]. These criteria thus cannot be expected to apply generally for the orthotropic case. The same prescribed orientation is given by the theoretical Norris equation, being far from wood behaviour that shows a zero, or nearly zero, slope of the ellipsoid with respect to the main direction. This explains why the older empirical Norris equation, with zero slope, that generally is applied for wood in Europe, is less worst than the theoretical Norris equation.
Although the Norris criterion is prescribed in the Codes for wood, (but not is applied as should be by four different equations for the four normal stress quadrants to account for the different strengths in tension and compression and the different fictive shear strengths), the flow rule according to the Hill-criterion is proposed for wood. More consistent this should have been based on the (corrected) Norris equations. However the only right results are obtained by the exact method that connects directly the strain rates of the general loading case to the deformation kinetics rate equations [6], being the physical constitutive equations for wood and other materials.

2. THE GENERAL FAILURE CRITERION FOR WOOD POLYMERS

2.1. General properties

A yield- or flow-criterion gives the combinations of stresses whereby flow occurs in an elastic-plastic material like wood in compression. For more brittle failure types in polymers with glassy components like wood at tensile loading, there is some boundary where above the gradual flow of components at peak stresses and micro-cracking may have a similar effect on stress redistribution as flow especially for long term loading. It is discussed in [10] and later that these flow and failure

boundaries may be regarded as equivalent elastic-plastic flow surfaces.
The flow- or failure criterion is a closed surface in the stress space i.e. a more dimensional space with coordinates $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6$.
A cut, e.g. according to figure 2.1 through the plane of the coordinate axes y = σ_1 and x = σ_2, will show a closed curve and such a curve always can be described by a polynomial in x and y like:

$$ax + by + cx^2 + dy^2 + exy + fx^3 + gy^3 + hx^2y + ixy^2 + ... = k \qquad (2.1)$$

Figure 2.1. Failure ellipsoid and definition of positive stresses.

whereby as much as terms can be accounted for as is necessary for the wanted precision. The surface will be concave because of the normality principle, and higher order terms, causing local peaks on the surface (and thus causing inflection points) are only possible by local hardening effects depending on the loading path and are outside the flow-criterion. These effects can be treated (purely descriptive) as given in [1] at: "2.2. Hardenings rules", or by the approach of [10].
It can also be seen that the constants f and g are indeterminate and have to be taken zero because, for y = 0, eq.(2.1) becomes: $ax + cx^2 + fx^3 = k$, having the real roots
$x_0,\ -x_1,\ -x_2$ and thus can be written:

$$(x - x_0)\cdot(x + x_1)\cdot(x + x_2) = 0 \qquad (2.2)$$

Because there are only two points of intersection possible of a closed surface with a line, there are only two roots by the intersecting x-axis e.g. $x = x_0$ and $x = -x_1$ and the part ($x + x_2$), being never zero within or on the surface and thus is indeterminate, has to be omitted. For a real concave surface f thus is necessarily zero.
The same applies for g: g = 0 following from the roots of y when x = 0.
The equation can systematically be written as stress-polynomial like:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k + ... = 1 \quad (i, j, k = 1, 2, 3, 4, 5, 6) \qquad (2.3)$$

In [1] it is shown that clear wood can be regarded to be orthotropic in the main planes and the principal directions of the strengths are orthogonal (showing the common tensor transformations) and higher order terms, that are due to hardening, normally can be neglected so that eq.(2.3) becomes:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \quad (i, j = 1, 2, 3, 4, 5, 6) \qquad (2.4)$$

In [10], and as discussed later, it is shown that this equation represents the critical distortional energy of failure.

In eq.(2.4) is, for reasons of energetic reciprocity, $F_{ij} = F_{ji}$ $(i \neq j)$ and by orthotropic symmetry in the main planes (through the main axes along the grain, tangential and radial) there is no difference in positive and negative shear-strength and terms with uneven powers in σ_6 thus are zero or: $F_{16} = F_{26} = F_6 = 0$; and there is no interaction between normal- and shear-strengths or: $F_{ij} = 0$ $(i \neq j;\quad i, j = 4, 5, 6)$.
Thus eq.(2.4) becomes for a plane stress state in a main plane:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \tag{2.5}$$

For a thermodynamic allowable criterion (positive finite strain energy) the values F_{ii} must be positive and the failure surface has to be closed and cannot be open-ended and thus the interaction terms are constrained to:

$$F_{11}F_{22} > F_{12}^2 \tag{2.6}$$

($F_{11}F_{22} = F_{12}^2$ gives a parabolic surface and $F_{11}F_{22} < F_{12}^2$ is hyperbolic)
For the uniaxial tensile strength $\sigma_1 = X$ $(\sigma_2 = \sigma_6 = 0)$ and eq.(2.5) becomes:

$$F_1\sigma_1 + F_{11}\sigma_1^2 = 1 \quad \text{or:} \quad F_1X + F_{11}X^2 = 1 \tag{2.7}$$

and for the compression strength $\sigma_1 = -X'$ this is:

$$-F_1X' + F_{11}X'^2 = 1 \tag{2.8}$$

and it follows from eq.(2.7) and (2.8) that F_1 and F_{11} are known:

$$F_1 = \frac{1}{X} - \frac{1}{X'} \quad \text{and} \quad F_{11} = \frac{1}{XX'} \tag{2.9}$$

In the same way is for $\sigma_1 = \sigma_6 = 0$, in the direction perpendicular:

$$F_2 = \frac{1}{Y} - \frac{1}{Y'} \quad \text{and} \quad F_{22} = \frac{1}{YY'} \tag{2.10}$$

Further it follows for $\sigma_1 = \sigma_2 = 0$ (pure shear), for the shear strength S, that:

$$F_{66} = \frac{1}{S^2} \tag{2.11}$$

and according to eq.(2.6) is: $-1/\sqrt{XX'YY'} < F_{12} < +1/\sqrt{XX'YY'}$ (2.12)
It can be shown (as discussed in [1]) that the restricted values of $2F_{12}$, based on assumed coupling according to the deviator stresses, as given by Norris [13], Hill or Hoffmann [14] as: $2F_{12} = -1/2XY$, or: $F_{12} = -(1/X^2 + 1/Y^2 - 1/Z^2)$ are not general enough for orthotropic materials and don't apply for wood. There also is no reason to restrict F_{12} according to e.g. Tsai and Hahn [15] as: $2F_{12} = 1/\sqrt{XX'YY'}$ or according to Wu and Stachurski [16] as: $2F_{12} \approx -2/XX'$. These chosen values suggest that $2F_{12}$ is ~ 0.2 to 0.5 times the extreme value of eq.(2.12).
The properties of a real physical surface in stress space have to be independent on the orientation of the axes and therefore the tensor transformations apply for the stresses σ of eq.(2.5). These transformation are derivable from the equilibrium of the stresses on an element formed by the rotated plane and on the original planes, or simply, by the analogous circle of Mohr construction. For the uniaxial tensile stress then is:

$\sigma_1 = \sigma_t \cos^2\theta \qquad \sigma_2 = \sigma_t \sin^2\theta \qquad \sigma_6 = \sigma_t \cos\theta\sin\theta$

Substitution in eq.(2.5) gives:

$$F_1\sigma_t\cos^2\theta + F_2\sigma_t\sin^2\theta + F_{11}\sigma_t^2\cos^4\theta + (2F_{12}+F_{66})\sigma_t^2\cos^2\theta\sin^2\theta + F_{22}\sigma_t^2\sin^4\theta = 1 \quad (2.13)$$

and substitution of the values of F:

$$\sigma_t\cos^2\theta\left(\frac{1}{X}-\frac{1}{X'}\right)+\sigma_t\sin^2\theta\left(\frac{1}{Y}-\frac{1}{Y'}\right)+\frac{\sigma_t^2\cos^4\theta}{XX'}+2F_{12}\sigma_t^2\sin^2\theta+\frac{\sigma_t^2\sin^4\theta}{YY'}+$$
$$+\frac{\sigma_t^2\cos^2\theta\sin^2\theta}{S^2}=1 \quad (2.14)$$

It can be seen that for θ = 0 this gives the tensile- and compression strength in e.g. the grain direction: $\sigma_t = X$ and $\sigma_t = -X'$, and for θ = 90⁰, the tensile and compression strength perpendicular to the grain: $\sigma_t = Y$ and $\sigma_t = -Y'$, and that a definition is given of the tensile and compression strengths in every direction. These are the points of intersection of the rotated axes with the failure surface. Eq.(2.13) thus can be read in this strength component along the rotated x-axis: $\sigma_t = \sigma_1$ according to: $F'_1\sigma_1 + F'_{11}\sigma_1^2 = 1$ (2.15)

and eq.(2.13) gives the definition of the transformations of F'_1 and F'_{11}. The same can be done for the other strengths. The transformation of F'_{ij} thus also is a tensor-transformation (of the fourth rank) that follows from the rotation of the symmetry axes of the material. Transformation thus is possible in two manners. The stress-components can be transformed to the symmetry directions according to eq.(2.5). Or the symmetry axes can be rotated, leaving the stresses along the rotating axes unchanged. For this case the general polynomial expression eq.(2.16) applies:

$$F'_1\sigma_1 + F'_2\sigma_2 + F'_{11}\sigma_1^2 + 2F'_{12}\sigma_1\sigma_2 + F'_{22}\sigma_2^2 + F'_{16}\sigma_1\sigma_6 + F'_{26}\sigma_2\sigma_6 + F'_{66}\sigma_6^2 = 1 \quad (2.16)$$

These transformations of F' are e.g. given in [1].

2.2. Initial yield criterion and derivation of the Hankinson and extended Hankinson equations

As will be discussed later, eq.(2.5) or eq.(2.14) for the off-grain-axis tensile- and compression strengths, represents the initial yield condition, being the extended critical distortional energy principle.

This "initial yield" equation, eq.(2.14), can be resolved into factors as follows:

$$\left(\frac{\sigma_t\cos^2\theta}{X}+\frac{\sigma_t\sin^2\theta}{Y}-1\right)\cdot\left(\frac{\sigma_t\cos^2\theta}{X'}+\frac{\sigma_t\sin^2\theta}{Y'}+1\right)=0 \quad (2.17)$$

giving the product of the Hankinson equations for tension and for compression, (where X and X′ are the strengths in grain direction). This is possible when:

$$2F_{12} + 1/S^2 = 1/X'Y + 1/XY' \quad (2.18)$$

In this equation, derived in [1], $(1/X'Y + 1/XY')$ is of the same order, and thus about equal to $1/S^2$ so that $2F_{12}$ is of lower order with respect to $1/S^2$. In [2]

eq.(2.18) was used as a measure for F_{12} what is a difference of two higher order quantities and thus can not give a precise information of the value of F_{12}, that also can be neglected as first estimate. In [5], wrongly the sum of $1/S^2$ and $(1/X'Y+1/XY')$ is taken to be equal to $2F_{12}$, being of higher order with respect to the real value of $2F_{12}$ and it evident that this value did not satisfy eq.(2.12).
Eq.(2.17) shows that the exponent "n" of the generalized Hankinson formula eq.(2.19):

$$\frac{\sigma_t \cos^n \theta}{X} + \frac{\sigma_t \sin^n \theta}{Y} = 1 \tag{2.19}$$

is: n = 2 for tension and compression at initial yield when there are no higher order terms. A value of n, different from n = 2 thus means that there are higher order terms due to hardening after initial yield as in eq.(2.21).
The initial yield criterion eq.(214) or eq.(2.17), being the extended critical distortional energy principle, should satisfy both the elastic and the yield conditions at the same time. Because the Hankinson equation with n = 2 also applies for the axial modules of elasticity and because this modulus is proportional to the strength, the Hankinson equations with n = 2, eq.(2.17), satisfies this requirement. Thus after some strain in the elastic stage, the initial yield is reached and because the modulus of elasticity follows the Hankinson equation with n = 2, also the yield criterion, eq.(2.17), containing the Hankinson equations, follows this and has the quadratic form and no higher order terms. This also is measured. It is mentioned in [8], that for glulam and for clear wood in bending and in tension, n $\approx$ 2. The combined compression with shear tests (of Keylwerth by the "Schereisen", allowing only shear-deformation in one plane) show that for off-axis longitudinal shear, also in the radial plane, n = 2, showing no higher order terms for the shear strength. According to fig. 2.3 this also applies for the tangential plane, but not for the radial plane. The value of n thus depends on the type of test and it is mentioned e.g. by Kollmann that n $\approx$ 2.5 for compression of clear wood, showing that hardening was possible in the tests and the third order terms of the yield criterion are not zero [10]. The test method of [4] shows that F_{112}, F_{166} and F_{266} for the radial plane have an influence, what is shown here to be the hardening effect due to crack arrest and due to confined dilatation. Thus the test method (early instability or not) has influence on whether only initial yield
(n = 2), or a more stable failure will occur (n different from n = 2). When n $\neq$ 2, higher order terms are not zero in the failure condition and eq.(2.21) applies.
An equation of the fourth degree (eq.(2.21) in σ_t) can always be written as the product of two quadratic equations, eq.(2.20). For a real failure surface the roots will be real and because the measurements show that one of the quadratic equations is determining for compression and the other for tension and must be valid for zero values of C_t and/or C_d as well, this factorization leads as the only possible solution to be the product of extended Hankinson equations for tension and compression as follows:

$$\left(\frac{\sigma_t\cos^2\theta}{X}+\frac{\sigma_t\sin^2\theta}{Y}-1+\sigma_t^2\sin^2\theta\cos^2\theta\cdot C_t\right)\cdot\Big(\frac{\sigma_t\cos^2\theta}{X'}+\frac{\sigma_t\sin^2\theta}{Y'}+1+$$

$$+\ \sigma_t^2\sin^2\theta\cdot\cos^2\theta\cdot C_d\Big)=0 \qquad (2.20)$$

Performing this multiplication, eq.(2.20) thus is in general:

$$F_1\sigma_t\cos^2\theta+F_2\sigma_t\sin^2\theta+F_{11}\sigma_t^2\cos^4\theta+\left(2F_{12}+F_{66}\right)\sigma_t^2\cos^2\theta+F_{22}\sigma_t^2\sin^4\theta+3\left(F_{112}+\right.$$

$$\left.+F_{166}\right)\sigma_t^3\cos^4\theta\cdot\sin^2\theta+3\left(F_{122}+F_{266}\right)\sigma_t^3\sin^4\theta\cdot\cos^2\theta+12F_{1266}\sigma_t^4\cos^4\sin^4\theta=1 \qquad (2.21)$$

giving the applied criterion in [4], where it appeared that F_{1122} and other possible higher order terms can be neglected except F_{1266}.

The values C_t and C_d can be found by fitting of the modified "Hankinson equations" eq.(2.20), for uniaxial off-axis tension and compression giving the constants:

$$2F_{12}=1/X'Y+1/XY'-1/S^2+C_t-C_d;\qquad 3\left(F_{112}+F_{166}\right)=C_t/X'+C_d/X;$$

$$3\left(F_{122}+F_{266}\right)=C_t/Y'+C_d/Y;\quad \text{and}\quad 12F_{1266}=C_tC_d-12F_{1122}\approx C_tC_d \qquad (2.22)$$

A fit of the Hankinson power equation, eq.(2.19) always is possible and different n values for tension and compression from n = 2 in that equation means that there are higher order terms and that C_t and C_d are not zero, as follows from eq.(2.20).

For timber with defects and grain and stress deviations, the axial strength is determined by combined shear and normal stress perpendicular to the grain. This may cause stable crack propagation and a parabolic curve of the effective shear strength (according to the Mohr- or Wu-equation, eq.(2.32) with c = 1) given by a third order term. For timber n can be as low as n $\approx$ 1.6 in eq.(2.19) for tension, showing higher order terms to be present. This also follows from n $\approx$ 2.5 for compression. The data of [4], show that F_{166}, F_{266} and F_{112} of the radial plane have influence what can be explained by an equivalent hardening effect due to crack arrest (see fig. 2.8, 2.7 and 2.6, showing parabolic like curves, different from elliptic curves of order 2 of fig. 2.5).

Thus the extended Hankinson equations, eq.(2.20) apply when in eq.(2.19) n < 2 for tension, and n > 2 for compression.

It was shown in [1] that F_{12} is small and can not be known with a high accuracy. Small errors in the strength values (X, X', Y, Y', S) may switch F_{12} from its lower bound to its upper bound, changing its sign and the value thus is not important and thus negligible for a first estimate. The data of [4] of the principal stresses in longitudinal tension, being close to initial yield, indicate F_{12} to be zero at initial yield, thus when $C_d = C_t = 0$ and thus when:

$$1/S^2=1/X'Y+1/XY' \qquad (2.23)$$

Then eq.(2.22) suggests that: $2F_{12}=C_t-C_d$ (2.24)

when C_t and C_d are not zero. This is tested here in the following and it appears that, because $F_{12}=0$ for longitudinal tension, S follows from:

$$1/S^2=1/X'Y+1/XY'+C_t-C_d$$

according to eq.(2.22) and S should not be measured separately by a different type of test, but follows from the uniaxial off-axis tension- and compression tests. Because F_{1122} is negligible, is, according to eq.(2.22): $12\,F_{1266} \approx C_t\,C_d$, what also is small and negligible, as will be shown later.
F_{166} will have a similar bound for F_{266} as follows from eq.(2.32) what is given in fig. 2.3, and follows by replacing the index 2 by 1 and Y by X. However the determining bound of F_{166} follows from eq.(2.22), when F_{112} is known. A general method to determine this bound of F_{112} is given in [1], (for F_{266}). For the purpose here, the following approach is sufficient:
The upper bound of F_{112} becomes determining when $\sigma_6 = 0$ and because for longitudinal compression, F_{122} is small (according to eq.(2.22)) because F_{266} dominates, the following equation applies:

$$\sigma_1\left(\frac{1}{X}-\frac{1}{X'}\right)+\sigma_2\left(\frac{1}{Y}-\frac{1}{Y'}\right)+\frac{\sigma_1^2}{XX'}+\frac{\sigma_2^2}{YY'}+3F_{112}\sigma_2\sigma_1^2=1 \tag{2.25}$$

This can be written:

$$\sigma_1\left(X'-X\right)+\sigma_1^2\left(1+3F_{112}\sigma_2XX'\right)=\left(1-\sigma_2/Y\right)\cdot\left(1+\sigma_2/Y'\right)\cdot XX' \tag{2.26}$$

The critical value of F_{112}, to just have a closed surface, will occur at high absolute values of σ_1 and σ_2, thus in the neighbourhood of $\sigma_1 \approx -X'$. Inserting safely this value in the smallest term of eq.(2.26) gives:

$$\sigma_1^2\left(1+3F_{112}\sigma_2XX'+\left(X'-X\right)/\left(-X'\right)\right)=\left(1-\sigma_2/Y\right)\cdot\left(1+\sigma_2/Y'\right)\cdot XX'$$

or: $$\frac{\sigma_1}{X'}=\sqrt{\frac{\left(1-\sigma_2/Y\right)\cdot\left(1+\sigma_2/Y'\right)}{1+c\sigma_2/Y'}} \qquad \text{where } c=3F_{112}Y'X'^2 \tag{2.27}$$

Thus when c = 1, the curve reduces to a parabola and the requirement to have a closed curve is c < 1. Thus: $3F_{112}<1/Y'X'^2$. (2.28)
More general when F_{12} and F_{122} are not negligible, the bound: c < 1 becomes:

$$c\approx 3F_{112}X'^2\,Y'-2F_{12}Y'X'+3F_{122}Y'^2\,X'<1 \tag{2.29}$$

for longitudinal compression, where besides $\sigma_1\approx -X'$, also $\sigma_2\approx -Y'$ is substituted in the contribution of the smallest term, as assumed determining point to just have a closed surface.
The same could be expected to apply for longitudinal tension, giving the same equation (2.27) with X′ replaced by X. However, because of an other type of failure, F_{112} and F_{122} are zero for longitudinal tension, see fig. 2.5 that is an ellipse, thus a second order equation, according to eq.(2.25) with $F_{112} = 0$.
The found (cut-off) parabola eq.(2.27) (for c close to c = 1) is, as eq.(2.32), equivalent to the Wu-fracture equation for shear with tension or compression perpendicular. For wood in longitudinal compression, this failure mechanism acts in the radial plane, giving high values of F_{266} and F_{112} close to their bounds of c ≈ 0.8 to 0.9. Because for wood with defects there always are deviations of the stress or of the grain from the regarded main directions, there always is combined shear-

normal stress loading in the real material planes where eq.(2.32) applies and F_{112} probably is an apparent value caused by F_{266} of the real inclined material planes. In order to have one overall criterion for the different failure types of longitudinal tension and compression and thus in order to connect the longitudinal tension region, where F_{112}, F_{12} and F_{122} are zero, when this region is separately fitted, to the longitudinal compression region, where F_{112} dominates, it is necessary to use the higher order terms as F_{1222}, F_{1222} and F_{1222}. These terms thus have no physical meaning and are due to the different types of failure in the different regions. For tension, the early instability of the test by splitting determines the strength, while for compression the late instability after hardening defines failure. It thus is necessary to fit both regions separately and not to apply one overall criterion.
With the estimates of F_{266} and F_{112} to be close to their bounds for compression, and with zero normal coupling terms for tension, all constants of the general failure criterion, eq.(2.21) are known, according to eq.(2.22), depending on C_d and C_t from uniaxial off-axis tension and compression tests.
Performing always the stress-transformation to the main planes, as done here, only simple transformation rules (circle of Mohr construction) have to be known for application.

2.3. Transverse strengths

In [1] it is shown that for rotations of the 3-axis, when this axis is chosen along the grain, eq.(2.5) and (2.16) may precisely describe the peculiar behaviour of the compression- tension- and (rolling) shear-strength perpendicular to the grain and the off-axis strengths without the need of higher order terms. The measured lines of the off-axis uniaxial transverse strength of fig. 2.2, follow precisely from eq.(2.15):

$$F'_1\sigma_1 + F'_{11}\sigma_1^2 = 1$$

When for compression the failure limit is taken to be the stress value after that the same, sufficient high, amount of flow strain has occurred, then the differences between radial- tangential- and off-axes strengths disappear and one, directional independent, strength value remains (see fig. 2.2). For tension perpendicular to the grain, only in a rather small region (around 90^0, see fig. 2.2) in the radial direction, the strength is higher and because in practice, the applied direction is not precisely known and needs not to be the weakest plane, a lower bound of the strength will apply that is independent of the direction. The choice of these limits means that:

$$F_1 - F_2 = 0 \text{ and } F_{11} - F_{22} = 0$$

and that also F_{12} is limited according to:

$$2F_{12} = F_{11} + F_{22} - F_{66}$$

Further then also is:

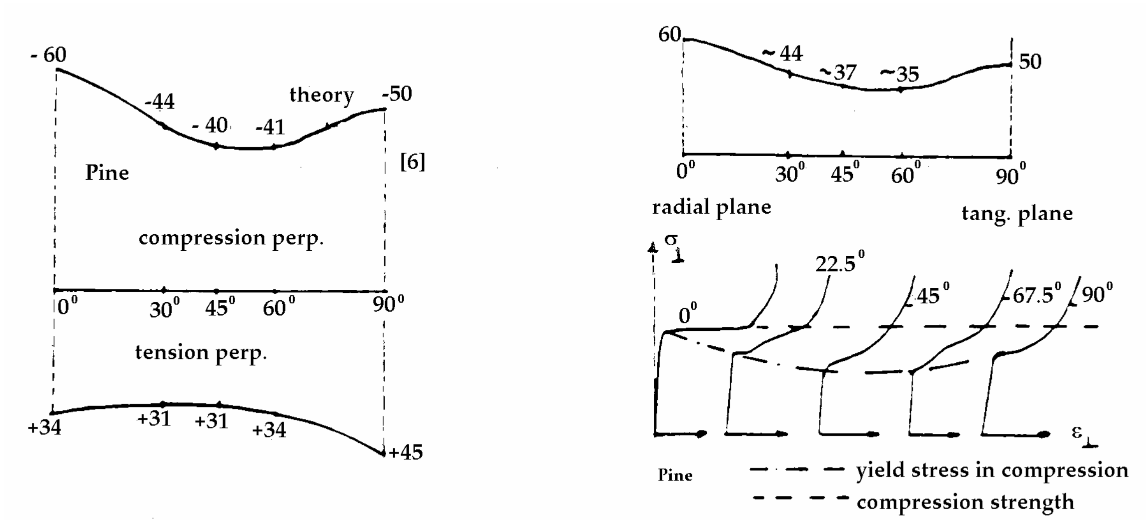


Figure 2.2 - Yield stresses and hardening

$$F'_6 = 0 \quad \text{and} \quad F'_{66} = F_{66} = 1/\tau_{rol}^2 \tag{2.30}$$

From measurement it can be derived that F_{12} is small leading to:

$F_{66} \approx F_{11} + F_{22}$ or τ_{rol} is bounded by:

$$\tau_{rol} = \sqrt{XX'/2} = \sqrt{YY'/2} \tag{2.31}$$

and the ultimate behaviour can be regarded to be quasi isotropic in the transverse direction.

The measurements further show for this rotation around the grain-axis that the "shear strengths" in grain direction in the radial- and the tangential plane, F_{44} and F_{55}, are uncoupled or $F_{45} = 0$, as is to be expected from symmetry considerations.

2.4. Longitudinal strengths

When now the 3-axis is chosen in the tangential or in the radial directions, the same relations apply (with indices 1, 2, 6) as in the previous case. The equations for this case then give the strengths along and perpendicular to the grain and the shear-strength in the grain direction.

In [1] it is shown that this longitudinal shear strength in the radial plane increases with compression perpendicular to this plane according to the coupling term F_{266} (direction 2 is the radial direction" direction 1 is in the grain direction):

$$F_2\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 3F_{266}\sigma_2\sigma_6^2 = 1 \quad \text{or:} \quad \frac{\sigma_6}{S} = \sqrt{\frac{(1-\sigma_2/Y)\cdot(1+\sigma_2/Y')}{1+c\sigma_2/Y'}} \tag{2.32}$$

with: c = $3F_{266}Y'S^2 \approx 0.9$ (0,8 to 0.99, see fig. 2.3).

When c approaches c $\approx$ 1 (measurements of project A in fig. 2.3), eq.(2.32) becomes:

$$\left(\frac{\sigma_6}{S}\right)^2 + \frac{\sigma_2}{Y} \approx 1 \tag{2.33}$$

being the Wu-equation of fracture mechanics for mixed mode I - II failure. This equation (2.33) can fully be explained by oriented micro-crack propagation [9]. As derived in [9], this equation does not only apply for tension with shear but also for shear with compression σ_2 perpendicular to the flat crack. For a high stress σ_2 the crack is closed at $\sigma_2 = \sigma_c$ and the crack tip notices only the influence of $\sigma_2 = \sigma_c$ because for the higher part of σ_2, the load is directly transmitted through the closed crack and eq.(2.33) becomes:

$$\frac{\sigma_6}{S} = \frac{-\mu(\sigma_2 - \sigma_c)}{S} + \sqrt{1 - \frac{\sigma_c}{Y}} \quad \text{or:} \quad \sigma_6 = C + \mu\,|\,\sigma_2\,| \tag{2.34}$$

where σ_2 and σ_c are negative, giving the Coulomb-equation with an increased shear capacity due to friction: $\mu\,|\,\sigma_2\,|$. However, inserting the measured values of [4], it appears that the frictional contribution is very small. The micro-crack closure stress σ_c will numerical be about equal to the tensile strength: $\sigma_c \approx$ - Y. The shear strength will be maximal raised, at high compression of $\sigma_c \approx$ - 0.9Y', by a factor:

$$\left(1 + \mu\left(0.9Y' - Y\right)/S\sqrt{2}\right) = \left(1 + 0.3\left(0.9 \cdot 5.6 - 3.7\right)/9.8 \cdot \sqrt{2}\right) = 1.03$$

Thus the shear strength with compression is mainly determined by an equivalent hardening effect, caused by crack arrest in the critical direction by the strong layers. At higher σ_2 stresses, compression plasticity perpendicular to the grain (project A of [11], see fig. 2.3), or instability of the test (project B of [11] with oblique-grain compression tests) may become determining, showing a lower value of c of eq.(2.32) than c = 1.
Because the slopes of the lines (at small σ_2) of project A and B of [11] are the same, there is no indication, for clear wood, of an influence of the higher order terms: F_{112}, F_{122} and F_{166} of project B. Further, the line of B is below the line of A and the c-value of B is lower, closer to the elliptic failure criterion. This is an indication that hardening after initial yield (thus departure from the elliptic equation) of project B, the oblique-grain compression test, is less than that of project A and thus that the test is less stable. (Project C of [11] follows the elliptic failure criterion because of the influence of transverse failure due to rolling shear that is shown before to be elliptic).
The bridging, crack arresting layers are not present in the tangential plane and because there thus is no influence of F_{355} in the tangential plane, what is the equivalent value of F_{266} of the radial plane, F_{266} of the radial plane diminishes quickly at axis-rotation (around the 2-axis), and this higher strength effect is only a local effect, only noticeable when loading is in the neighbourhood of the radial plane.
The high value of F_{266} (measured with $\sigma_1 = 0$), indicates that for clear wood, F_{122} will be negligible in the radial plane according to eq.(2.22). It also follows from published Hankinson lines of clear wood that F_{122} and F_{266} may be zero in the tangential plane, confirming the results of projects A and B of [11], mentioned before. The Hankinson lines with n $\approx$ 2 in eq.(2.19), show all higher order terms to be

zero. There is an indication that this is a general property of timber [11], because when shear failure is free to occur in the weakest plane, as usually in large timber beams and glulam, it occurs in the tangential plane and n = 2, showing no higher order terms.

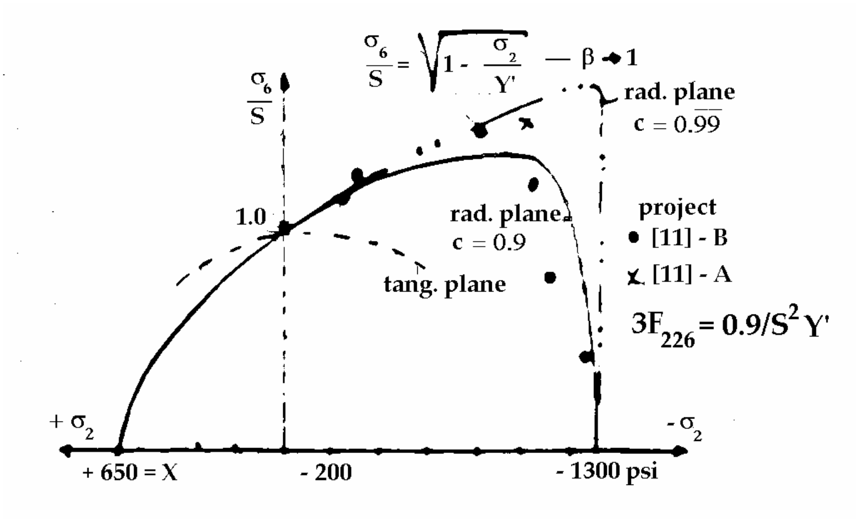


Figure 2.3 - Combined shear-tension and shear-compression strengths.

2.5. Estimation of the polynomial constants by uniaxial tests

Based on data fitting of uniaxial tension- and compression tests of [4], the values of C_d and C_t are determined and by eq.(2.22) the polynomial constants. This is compared with the data and fit of the biaxial measurements of [4].
In fig. 2.4, a determination of C_d and of C_t is given. In this figure of [4], the compression- strength perpendicular to the grain measurement Y′/X′ = 0.204 is reduced to obtain a value of Y′/X′ = 0.13 (at 90^0) to be able to use the measured constants of the bi-axial tests. It is not mentioned how that possibly can be done but the drawn lines in the figure give the prediction of the uniaxial values based on the measured constants according to the general eq.(2.21) (given in [4] in the strength tensor form as given here by eq.(2.15)). For comparison the fits of the Hankinson equations are given following these drawn lines.
For tension the extended Hankinson equation (2.20) becomes (by scratching the non zero term of the product:

$$\frac{\sigma_t \cos^2\theta}{X} + \frac{\sigma_t \sin^2\theta}{Y} + \sigma_t^2 \sin^2\theta \cos^2\theta \cdot C_t = 1 \qquad (2.35)$$

and this equation fits the line for tension in fig. 2.4 when $C_t \approx 11.9/X^2$. The Hankinson equation (2.19) fits in this case for n ≈ 1.8 and all 3 equations (2.21), (2.35) and (2.19) give the same result although for the Hankinson equations only the main tension- and compression strength have to be known and the influence of all other quantities are given by: n or by C_t.

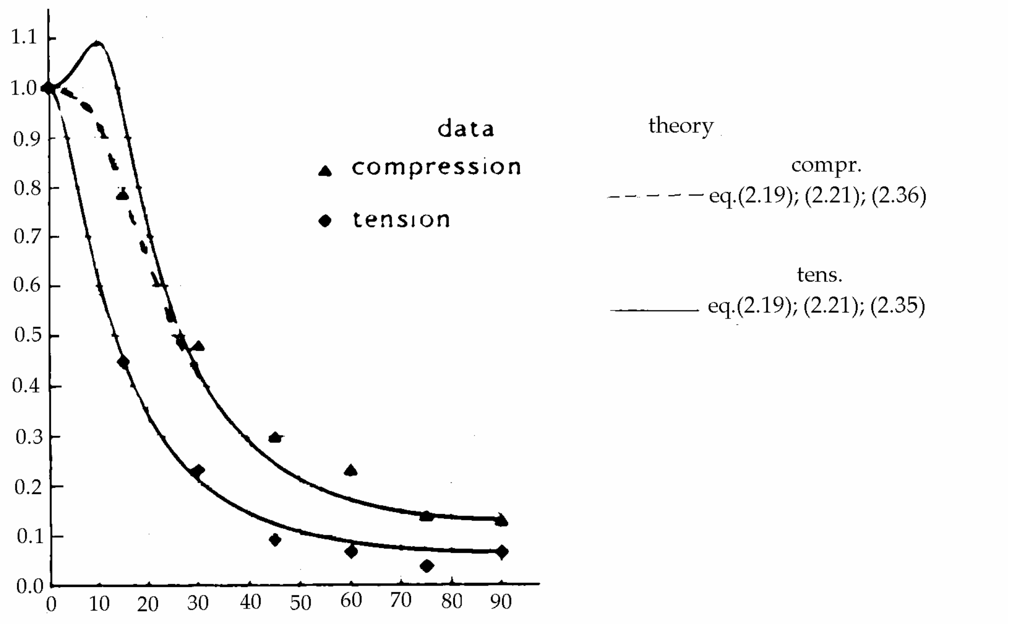


Figure 2.4 - Uniaxial tension- and compression strengths

For compression, the same line as found in [4] was found in [1], (see fig. 11 of [1]), by the second order polynomial with the minimal possible value of F_{12} (according to eq.(2.12)), showing that except a negative F_{122} (as used in [4]) also a high negative value of F_{12} may cause the strong peak at small angles. Because such a peak never is measured, the drawn line of [4] is only followed here for the higher angles by the Hankinson equation. For the small angles, the line (dashed) is drawn through the measured point at 15^0, giving the expectable Hankinson value of $n = 2.4$ in eq.(2.19) and for eq.(2.36): $C_d \approx 4/X'^2$. Because of this low measured value, the predicted peak at 10^0 in fig. 2.4 is not probable, although the peak-factor of 1.1 is theoretically possible, for high shear strength, to occur at 18^0 in stead of 10^0 with $C_d \approx 7.6/X'^2$ in the extended Hankinson equation (2.36):

$$\frac{\sigma_t \cos^2\theta}{X'} + \frac{\sigma_t \sin^2\theta}{Y'} + \sigma_t^2 \sin^2\theta \cos^2\theta \cdot C_d = 1 \tag{2.36}$$

This shows that the fit of the polynomial constants, based on the best fit of the measurements of [4], is not well for the oblique grain test. The explanation of this deviation is probably the different state of hardening of the data that can be more or less strong, depending on the stability of the type of test what is less in the Hankinson test. This, for instance, follows from the ratio of the compression strengths perpendicular to the grain and along the grain of 0.2 in the uniaxial tests and 0.1 in the biaxial tests showing more hardening in the biaxial tests. Further the strong local peak is never measured in the common oblique grain test, showing less stability than in the biaxial test.

An analogous behaviour occurs in the oblique grain test of clear wood where the

tensile test shows $C_t = 0$ in eq.(2.20) and the compression test may show C_d to be not zero. A zero value of C_t indicates no higher order terms and thus C_d should be zero. However the tensile test shows unstable failure at yield what needs not to be so for the compression test that may show additional hardening. Thus the criterion eq.(2.20) with only $C_t = 0$ may show two different hardening states. For the different hardening states in the different possible types of tests, the lowest always possible value should be used for practice thus $C_t = C_d = 0$.
It thus has to be concluded that the strong hardening in the biaxial test will not occur in all circumstances and the hardening parameters should be omitted for a safe lower bound criterion (in accordance with the oblique grain test).
As generally found in [1] for spruce clear wood, a fit is possible for off-axis tension by a second order polynomial with F_{12} = 0. This also applies for wood with defects, as follows from a fit of the data of [4] by the second order polynomial (ellipse) in the principal stresses σ_1 and σ_2 (σ_6 = 0), for longitudinal tension (σ_1 > 0; $F_{12} = 0$), see fig. 2.5. This means that F_{112} and F_{112} are zero (for σ_1 > 0) in the radial plane and because the Hankinson value for tension n is different from n = 2, there

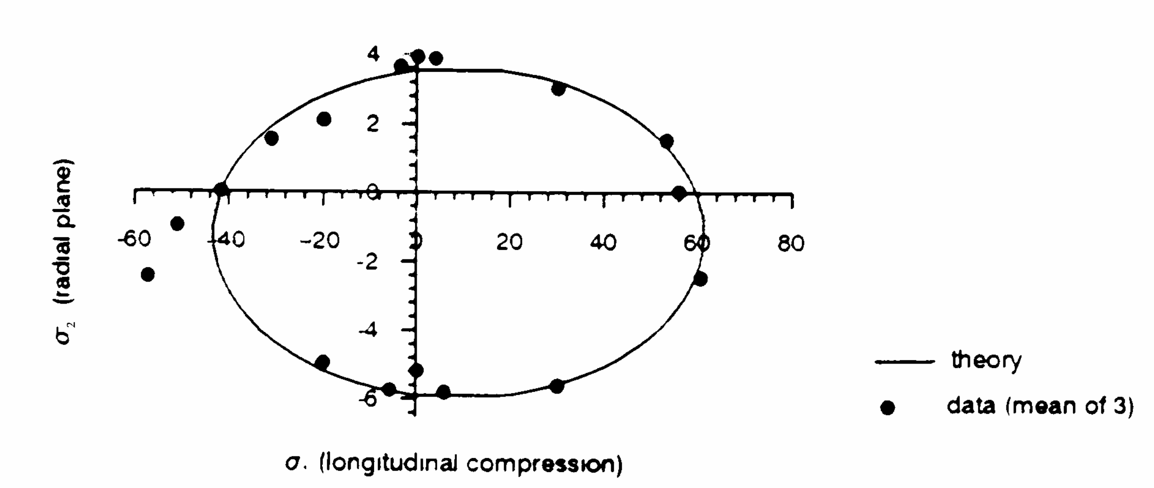


Figure 2.5 – First yield criterion eq.(2.5), with $F_{12} = 0$, for σ_6 = 0.

must be higher order terms for shear (F_{166}, F_{266}).
A first hypothesis thus is by eq.(2.22):
$3F_{266} = C_t / Y'$; $3F_{166} = C_t / X'$, (with $F_{112} = F_{122} = 0$) for tension and:
$3F_{166} = C_d / Y$ and $3F_{122} = C_d / X$, (with $F_{166} = F_{266} = 0$) for compression.
This assumption gives maximal values for F_{122} and F_{112} for the total fit of all data.
The strength values according to this fit of [4] are (in N/mm^2):
X = 59.5; X′ = 46.5; Y = 3.5; Y′ – 5.9: S ≈ 10 and with: $C_t = 11.9 / X^2$; $C_d = 4 / X'^2$;
$2F_{12} \approx C_d - C_t$, the predicted values are given in table 1 at column: hyp.1. It is seen that these values fit better than the best values of the comparable eq.(62) of [4], given in the column indicated with [4]. However for σ_6 = 0, F_{122} must be negative for a precise fit when σ_1< 0 and about zero when σ_1 > 0, showing that F_{122} has got

the function to replace neglected still higher order terms, and a precise fit only can be expected to be possible with multiple higher order terms (with indices 1 and 2). As mentioned in [4], the values of σ_6 can be corrected by F_{1266} to be slightly lower when the sign of σ_1 and σ_2 is the same and to be slightly higher when the sign is opposite. This means that the first and third row-value of column: hyp.1 (being 1.1 and 1.0) can be around 1.05. This shows that introduction of F_{1266} only gives a local correction of a few percent and justifies the negligence of F_{1266}. The column values further are slightly too high when $\sigma_1 > 0$ and too low for $\sigma_1 < 0$, indicating that F_{166} is not precise. Neglecting the multiple higher order terms, the hypothesis has to be rejected, because F_{122} is too high, distorting the ellipse (at $\sigma_1 > 0$) too much for high negative values of σ_2 and causing the surface to be open at $\sigma_1 < 0$ and at high negative σ_2. It thus has to be concluded that C_t and C_d are coupling terms between tension and compression and that the different types of failure in longitudinal tension and in compression should be given in separate failure criteria for these cases. Without the other higher order terms, F_{122} has to satisfy eq.(2.28) and the highest possible positive value of F_{122} becomes about 0.0001, being 5 times smaller than according to the first hypothesis. The fit now, with this small positive value of F_{122}, is about comparable with the best fit of [4] (that is based on a negative value of F_{122}), but now satisfies eq.(2.22) and will not show the compression peak in the Hankinson test (fig. 2.4). The fit is in total not better than a fit with changed constants and is also in total not better than assuming F_{12}, F_{122} and F_{112} to be zero for $\sigma_1 > 0$. This leads to the second hypothesis, that the higher order terms for normal stresses are small at fracture (close to initial yield behaviour when $\sigma_1 >$ 0) and thus can be neglected for a criterion in practice.

In table 1, column hyp. 2, the fit is given for $F_{12} = F_{112} = F_{122} = 0$. Because the fit does not change much when data above the uniaxial compression strength: X′ = 41.7 are neglected, the fit is based on this value and column hyp.2 thus gives the prediction of failure by the same hardening state as in the oblique grain test (where the strong compression hardening does not occur). The constants are:

$C_t = 11.9/X^2 = 11.9/59.5^2 = 0.00336;\quad C_d = 4/X'^2 = 4/41.7^2 = 0.00230$ and by eq.(2.22):

$3F_{266} = C_t/Y' + C_d/Y = 0.00332/5.95 + 0.0023/3.5 = 0.00122$ or c of eq.(2.32) is:

$c_{266} = 0.00122 \cdot 9.7^2 \cdot 5.95 = 0.68$ and:

$3F_{166} = C_t/X' + C_d/X = 0.00336/41.7 + 0.0023/59.5 = 0.000119$, or:

$c_{166} = 0.000119 \cdot 9.7^2 \cdot 41.7 = 0.47$.

$F_1 = \dfrac{1}{X} - \dfrac{1}{X'} = 1/59.5 - 1/41.7 = -0.0072;\qquad F_{11} = \dfrac{1}{XX'} = 1/(59.5 \cdot 41.7) = 0.00040,$

$F_2 = \dfrac{1}{Y} - \dfrac{1}{Y'} = 1/3.5 - 1/5.95 = 0.092;\qquad F_{22} = \dfrac{1}{YY'} = 1/(3.5 \cdot 5.95) = 0.048$ and:

$F_{66} = \dfrac{1}{S^2} = 1/9.7^2 = 0.0106;\qquad F_{12} = F_{112} = F_{122} = 0.$

This also applies as a lower bound for longitudinal compression (see fig. 2.5). For longitudinal compression, the torsion tube test for the principal stresses (σ_6 = 0; σ_1 < o) shows a parabolic curve in the radial plane, close to the Wu-equation, showing F_{112} to be high. This parabolic curve is given in fig. 2.6.

For comparison of the results, the strength values of the best fit of all data of [4] are regarded for longitudinal compression:

X = 55.5; X' = 43.1; Y = 3.7 ; Y' = 5.6. The shear strength S = 10 is too high, as follows from:

$1/S^2 \approx 1/X'Y + 1/XY' + C_t - C_d - 2F_{12} =$

$1/(43.1\cdot 3.7) + 1/(55.5\cdot 5.6) + 0.00386 - 0.00215 + 0.0014 = 0.0119,$

giving S ≈ 9.4. For the constants now is:

$F_1 = \dfrac{1}{X} - \dfrac{1}{X'} = 1/55.5 - 1/43.1 = -0.0052;\quad F_{11} = \dfrac{1}{XX'} = 1/(55.5\cdot 43.1) = 0.00042;$

$F_2 = \dfrac{1}{Y} - \dfrac{1}{Y'} = 1/3.7 - 1/5.6 = 0.092;\quad F_{22} = \dfrac{1}{YY'} = 1/(3.7\cdot 5.6) = 0.048.$

Further is:

$C_t = 11.9/X^2 = 11.9/55.5^2 = 0.00386$ and $C_d = 4/X'^2 = 4/43.1^2 = 0.00215.$

As mentioned, F_{266} and F_{112} will be high, close to their bonds giving:

$3F_{266} = 0.9/S^2Y' = 0.9/(9.4^2\cdot 5.6) = 0.00184$

and according to eq.(2.22):

$3F_{122} = C_t/Y' + C_d/Y - 3F_{266} = 0.00386/5.6 + 0.00215/3.7 - 0.00184 = -\,0.00057$

$3F_{112} = 0.9/(5.6\cdot 43.1^2) = 0.000086$ and consequently $3F_{166} = 0.000042$ with:

$2F_{12} = -\,0.0014$.

This gives the best fit for σ_6 = 0. However for combined shear with normal stresses, given in table 1, column 3-compr. the values are comparable with those of column [4] and not well enough, also not for practice.

A better fit for the shear strength is obtained by a slightly reduced factor 0.8 in stead of 0.9 for F_{112}, thus with less hardening, similar to the oblique grain test proj. B of fig. 2.3, giving:

$3F_{122} = 0.8/(5.6\cdot 43.1) = 0.000077;$

$3F_{166} = C_t/X' + C_d/X - 3F_{122} = 0.000128 - 0.000077 = 0.000051,$

giving the c-values: $c_{166} = 0.000051\cdot 9.4^2\cdot 43.1 = 0.2$ and: $c_{266} = 0.9$ (starting point).

These combined shear strengths are given in table 1, column 4 (compression fit), and it is seen that the fit is better than the foregoing column. For σ_6 = 0, the fit is given in fig. 2.6 for compression.

For longitudinal compression eq.(2.21) becomes:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 3F_{112}\sigma_1^2\sigma_2 + 3F_{122}\sigma_2^2\sigma_1 +$$
$$+ 3F_{166}\sigma_6^2\sigma_1 + 3F_{266}\sigma_6^2\sigma_2 = 1 \qquad (2.37)$$

Because the C_t, C_d and n-values of the Hankinson equations are sufficiently close to the published extreme values of n, the here calculated c-values can be used in

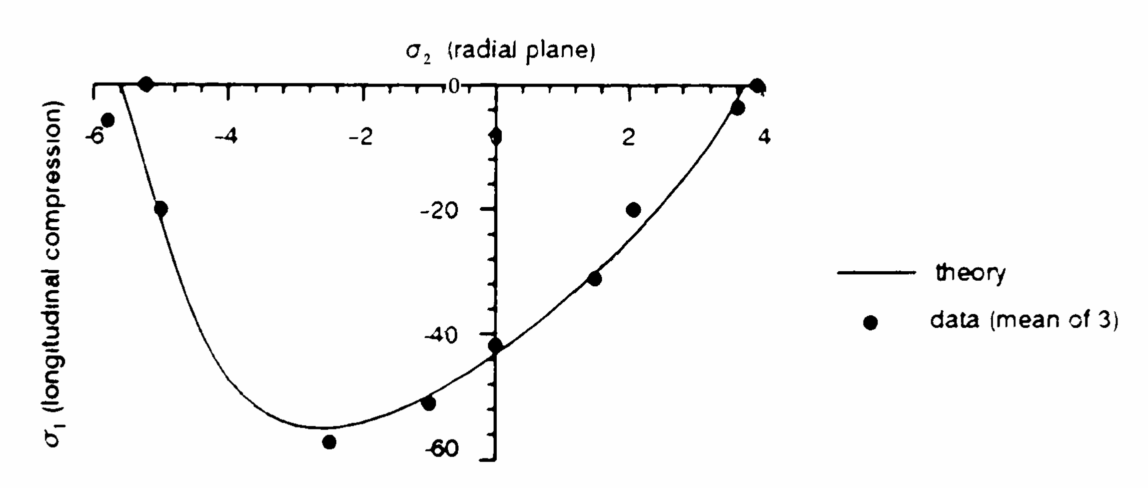


Figure 2.6 – Yield criterion for compression ($\sigma_1 < 0$) for $\sigma_6 = 0$.

Table 1. Shear strength σ_6 for combined normal stresses

σ_1	σ_2	σ_6 test	factor: $\sigma_{6,\text{theory}} / \sigma_{6,\text{test}}$				
			[4]	hyp. 1	hyp. 2 tens.	3 compr.	4 compr.
30	1.5	5.8	1.07	1.10	1.03	0.90	1.02
30	0	8.5	0.88	0.97	0.91	0.77	0.92
30	-2.5	7.9	0.99	1.00	1.10	0.91	1.29
7.3	0	9.2	1.04	1.07	1.03	0.96	1.01
0	2.9	3.7	1.38	1.25	1.13	1.39	1.19
0	1.5	8.5	0.96	0.95	0.89	0.93	0.86
0	0	9.0	1.11	1.11	1.08	1.04	1.04
0	-2.5	10.9	0.96	0.98	1.05	0.86	1.07
0	-5.4	6.8	0.53	0.82	1.12	0.45	1.12
-7.7	0	9.6	1.05	1.04	1.01	1.03	0.96
-20	1.5	7.7	0.84	0.89	0.83	0.93	0.68
-20	0	9.6	0.99	0.98	0.96	1.10	0.88
-30	-2.5	11.3	1.04	0.98	0.90	1.16	0.94
mean factor			0.99	1.01	1.0	0.96	1.0

general and inserting F-values in eq.(2.37), this equation becomes:

$$\frac{\sigma_6^2}{S^2}\cdot\left(1+0.9\cdot\frac{\sigma_2}{Y'}+0.2\cdot\frac{\sigma_1}{X'}\right)=\left(1-\frac{\sigma_1}{X}\right)\cdot\left(1+\frac{\sigma_1}{X'}\right)+\left(1-\frac{\sigma_2}{Y}\right)\cdot\left(1+\frac{\sigma_2}{Y'}\right)+$$

$$-\left(1+0.8\cdot\frac{\sigma_2\sigma_1^2}{Y'X'^2}-0.77\cdot\frac{\sigma_1\sigma_2^2}{X'Y'^2}-0.41\cdot\frac{\sigma_1\sigma_2}{X'Y'}\right) \tag{2.38}$$

For longitudinal tension ($\sigma_1 \geq 0$), eq.(2.21) becomes:

$$\frac{\sigma_6^2}{S^2}\cdot\left(1+0.68\cdot\frac{\sigma_2}{Y'}+0.47\cdot\frac{\sigma_1}{X'}\right)=\left(1-\frac{\sigma_1}{X}\right)\cdot\left(1+\frac{\sigma_1}{X'}\right)+\left(1-\frac{\sigma_2}{Y}\right)\cdot\left(1+\frac{\sigma_2}{Y'}\right)-1 \tag{2.39}$$

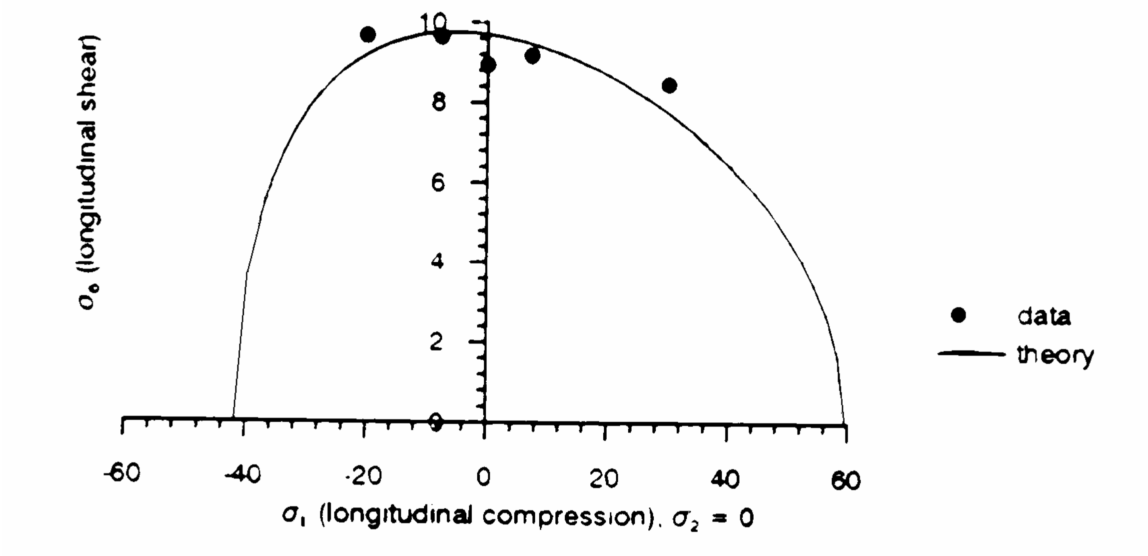


Figure 2.7 – Combined longitudinal shear with normal stress in grain direction.

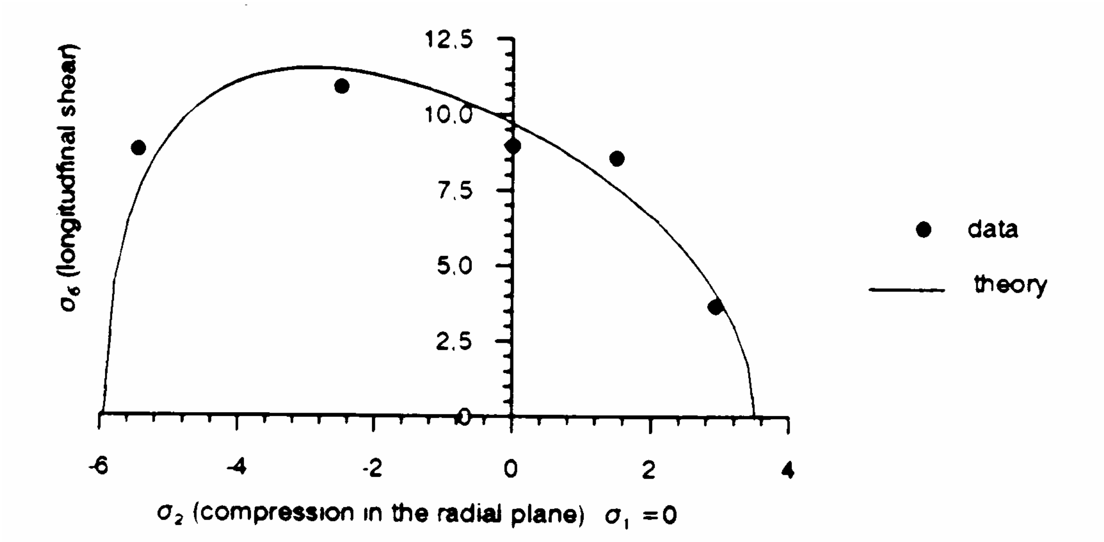


Figure 2.8 - Longitudinal shear strength ($\sigma_1 = 0$) depending on the normal stress.

Because the compression hardening according to eq.(2.38) occurs for low values of σ_6 only, and only in the torsion tube test in the radial plane, eq.(2.39) more generally represents the failure criterion for both tension and compression for the more common failure case when $n \neq 2$. Neglecting the local higher compression

strengths in the radial plane above the uniaxial compression strength, at $\sigma_6 = 0$, also the overall fit by this equation is very well and even better than the proposed fit of [4].
For tests and structures, showing early instability at failure, the higher order terms may be zero, causing the Hankinson value of n = 2 for timber and glulam. Because this is to be expected in most situations, the determining criterion becomes:

$$\frac{\sigma_6^2}{S^2} - \left(1 - \frac{\sigma_1}{X}\right)\cdot\left(1 + \frac{\sigma_1}{X'}\right) - \left(1 - \frac{\sigma_2}{Y}\right)\cdot\left(1 + \frac{\sigma_2}{Y'}\right) = -1,$$

or worked out, identical to eq.(2.5) with $F_{12} = 0$:

$$\frac{\sigma_6^2}{S^2} + \frac{\sigma_1}{X} - \frac{\sigma_1}{X'} + \frac{\sigma_1^2}{XX'} + \frac{\sigma_2}{Y} - \frac{\sigma_2}{Y'} + \frac{\sigma_2^2}{YY'} = 1 \tag{2.40}$$

It therefore is necessary to use eq.(2.40) in the Codes in all cases for timber and clear wood to replace the now commonly used, not valid Norris-equations.
This criterion is a critical strain energy condition of the reinforcements leading to eq.(3.9) for equal tension and compression strengths and to eq.(3.11) with $F_{12} = 0$, for wood. Because the increase of the strain energy of a loaded body due to crack extension is equal to the apparent surface energy α per unit area of a crack with area ct, this condition:
$\frac{\partial}{\partial c}\left(W - 4\alpha ct\right) = 0$ leads to: $W_0 = \alpha/\left(\pi c\right)$, where W_0 is strain energy of the uncracked body. For uniaxial tension this is: $\sigma_t^2/2E = \alpha/\left(\pi c\right)$, giving the Griffith strength: $\sigma_t = \sqrt{2E\alpha/\pi c}$
In this case here, $\Sigma W_0/(\alpha/\left(\pi c\right)) = 1$ gives eq.(2.40). This only is possible when slip of the reinforcement due to cracking, occurs at all stress levels and when there are 2 interacting crack systems in the 2 reinforcement directions for plane stress, that interact for failure.

3. DERIVATION OF THE CRITICAL DISTORSIONAL ENERGY PRINCIPLE

3.1 Yield criterion.

A yield- or flow-criterion gives the combinations of stresses whereby flow occurs in an elastic-plastic material. For more brittle failure types in polymers with glassy components like wood at tensile loading, there is some boundary where below the behaviour is assumed to be elastic and where above the gradual flow of components at peak stresses and micro-cracking may have a similar effect as plastic flow with hardening (like metals with gradual plasticity and no yield point).
The loading, damage and hardening behaviour up to failure can fully be described by deformation kinetics [6]. There are several processes acting causing early local

flow and stable micro-crack propagation, while the main part of the material is elastic and different inelastic strain rate equations are necessary depending on the loading type and material zone.
The failure criterion depends on the ultimate damage process and failure occurs when the standard test becomes unstable.

3.2. Critical distortional energy

The initial yield criterion gives the boundary where below the behaviour is elastic. When loaded very quickly this boundary also can be high, not very far from the by standard tests defined failure surface and the criterion may also represent a lower bound of the failure criterion. The same applies for wood with small defects, showing early failure. The axial loading behaviour then often can be regarded to be linear elastic up to failure.
Because an isotropic matrix of a material may sustain large hydrostatic pressures without yielding, yield can be expected to depend on a critical value of the distortional energy. This energy is found by subtracting the energy of the volume change from the strain energy. Thus for an isotropic material this is:

$$\left(\frac{1}{2E}\cdot\left(\sigma_x^2+\sigma_y^2+\sigma_z^2\right)-\frac{\nu}{E}\cdot\left(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x\right)+\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2\right)\right)+$$
$$-\left(\frac{1-2\nu}{6E}\cdot\left(\sigma_x+\sigma_y+\sigma_z\right)^2\right)=$$
$$=\frac{1+\nu}{6E}\cdot\left(\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2\right)+\frac{1}{2G}\cdot\left(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2\right) \tag{3.1}$$

For plane stress, the distortional energy thus is with $2G = E/(1+\nu)$:

$$\frac{1+\nu}{3E}\cdot\left(\sigma_x^2-\sigma_x\sigma_y+\sigma_y^2+3\tau^2\right) \tag{3.2}$$

When σ_x, σ_y and τ are the nominal stresses of a material, having a reinforcement in x and y direction that takes a part of the loading, then the distortional energy of the matrix becomes:

$$\frac{1+\nu}{3E}\cdot\left(\left(1-c_x\right)\sigma_x^2-\sigma_x\sigma_y+\left(1-c_y\right)\sigma_y^2+3\left(1-c_{tx}-c_{ty}\right)\tau^2\right), \tag{3.3}$$

where the reinforcement parts are subtracted from the total load. For the reinforcement, taking only normal force and shear , this is:

$$\frac{1+\nu'}{3E_a}\cdot\left(\sigma_{ax}^2+3\tau_a^2\right)\quad \text{with:}\quad \sigma_{ax}=\left(\frac{E_a}{E}-1\right)\cdot\omega_x\sigma_x$$

where ω_x is the area of the reinforcement per unit area, giving:

$$c_x=\frac{1+\nu'}{1+\nu}\cdot\left(1-\frac{E}{E_a}\right)^2\cdot\omega_x^2\cdot\frac{E_a}{E} \tag{3.4}$$

The other values of c_i are analogous.
When the distortional energy is constant at yield then eq.(3.3) gives:

$$\left(1-c_x\right)\sigma_x^2-\sigma_x\sigma_y+\left(1-c_y\right)\sigma_y^2+3\left(1-c_{tx}-c_{ty}\right)\tau^2=C \tag{3.5}$$

For $\sigma_y=\tau=0$, this gives the yield stress in x-direction $\sigma_x=X'$. In the same way $\sigma_y=Y'$, when $\sigma_x=\tau=0$ and is $\tau = S$ when $\sigma_x=\sigma_y=0$, giving the equation:

$$\frac{\sigma_x^2}{X'^2}-2F_{12}\sigma_x\sigma_y+\frac{\sigma_y^2}{Y'^2}+\frac{\tau^2}{S^2}=1 \tag{3.6}$$

The Norris equation follows from eq.(3.6) when $2F_{12}=1/X'Y'$. This however is a special value of $2F_{12}$ that need not to apply in general.

For the special case that: $c_{tx}=c_{ty}=0$, and when, as for concrete, it is assumed that the reinforcement takes no shear, eq.(3.5) becomes:

$$\frac{\sigma_x^2}{X'^2}-\frac{\sigma_x\sigma_y}{3S^2}+\frac{\sigma_y^2}{Y'^2}+\frac{\tau^2}{S^2}=1 \tag{3.7}$$

and because $3S^2 \approx X'Y'$, as applies for isotropy what is assumed for the cell walls by Norris in his derivation and as also is measured, this equation becomes:

$$\frac{\sigma_x^2}{X'^2}-\frac{\sigma_x\sigma_y}{X'Y'}+\frac{\sigma_y^2}{Y'^2}+\frac{\tau^2}{S^2}=1 \tag{3.8}$$

giving the Norris equation as critical distortional energy equation of the matrix when the reinforcement "flows" and thus only may carry a normal force.

At early failure of the matrix, the reinforcement carries the total load by the normal- and shear forces and the coupling term disappears and the equation gives the apparent critical distortional energy of the reinforcement:

$$\frac{\sigma_x^2}{X'^2}+\frac{\sigma_y^2}{Y'^2}+\frac{\tau^2}{S^2}=1 \tag{3.9}$$

being the older empirical Norris equation.

The Norris equations (3.8) and (3.9) give the possible extremes of F_{12} between zero

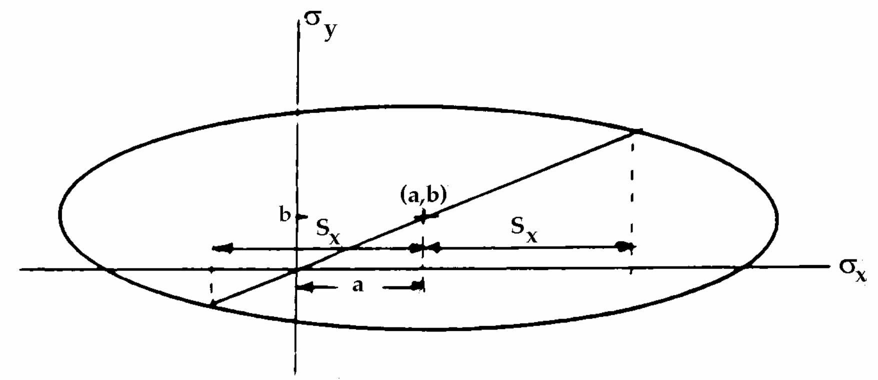


Figure 3.1. - Initial yield, von Mises criterion for wood.

and the maximal value. Although the Norris-equations are used for wood, they only apply for materials with equal compression and tension strengths.

When these yield strengths are not equal, different critical distortional energies will apply for tension and compression.

When a straight loading line (proportional loading) intersects the yield criterion at σ_{x1}, σ_{y1}, (both positive) in the first stress quadrant, it intersects the yield criterion in the third quadrant at σ_{x2}, σ_{y2} (both negative). Writing $\sigma_{x1} = a + s_x$; $\sigma_{y1} = b + s_y$ $\sigma_{x2} = a - s_x$; $\sigma_{y2} = b - s_y$, eq.(3.5) gives for positive stresses, with: $1 - c_x = c_x'$, etc.:

$$c_x'(a+s_x)^2 - (a+s_x)\cdot(b+s_y) + c_y'\cdot(b+s_y)^2 + c_t\tau^2 = C_1$$

and for negative stresses:

$$c_x'(a-s_x)^2 - (a-s_x)\cdot(b-s_y) + c_y'\cdot(b-s_y)^2 + c_t\tau^2 = C_2$$

Summation of both critical distortion energies C_1 and C_2, gives the total energy of both events and gives the collection or all critical points, thus gives the equation of the yield surface, thus:

$$c_x'\cdot s_x^2 - s_x s_y + c_y'\cdot s_y^2 + c_t\tau^2 = C_3. \tag{3.10}$$

Substitution of $s_x = \sigma_{x1} - a$ and $s_y = \sigma_{y1} - b$, or: $- s_x = \sigma_{x2} - a$ and $- s_y = \sigma_{y1} - b$, leads to the general polynomial [1]:

$$\frac{\sigma_x^2}{XX'} + \frac{\sigma_x}{X} - \frac{\sigma_x}{X'} + 2F_{12}\sigma_x\sigma_y + \frac{\sigma_y^2}{YY'} + \frac{\sigma_y}{Y} - \frac{\sigma_y}{Y'} + \frac{\tau^2}{S^2} = 1 \tag{3.11}$$

or with $\sigma_x = \sigma_1; \sigma_y = \sigma_2$ and $\tau = \sigma_6$:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \tag{3.12}$$

as the critical distortional energy criterion in the main planes determining first yield. The derivation is of course not essentially different for an orthotropic reinforcement. Due to the system there is an equivalent slip of the reinforcement for shear loading for wood and G is not coupled according to 2G = E/(1 + ν) but a separate value of G has to be maintained. This also has no influence on the derivation.

3.3 Hankinson equations

The Hankinson equations apply for the off-axis uniaxial strengths and has to satisfy eq.(3.11) for initial yield. For the uniaxial tensile stress is:

$$\sigma_1 = \sigma_t\cos^2\theta \qquad \sigma_2 = \sigma_t\sin^2\theta \qquad \sigma_6 = \sigma_t\cos\theta\sin\theta$$

Substitution of these stresses in eq.(3.11) gives eq.(2.14)
This initial yield equation, eq.(2.14), can be resolved into factors giving eq.(2.17), what is the product of the Hankinson equation for tension and for compression. As discussed before, this is possible because according to eq.(2.18):

$$2F_{12} + 1/S^2 \approx 1/X'Y + 1/XY'$$

In the generalized Hankinson equation, eq.(2.19):

$$\frac{\sigma_t\cos^n\theta}{X} + \frac{\sigma_t\sin^n\theta}{Y} = 1$$

is the exponent n = 2 for the initial yield equation. Measured is also n = 2 for the strengths in bending and in tension of clear wood, also for veneer and for shear in the radial plane measured with the "Schereisen"-device. The measurements thus indicate that also in the radial plane n = 2 applies for initial yield. For $n \neq 2$, as may apply for compression, the extended Hankinson equations, eq.(2.20), apply.

3.4. Rankine criterion 25

The Hankinson equation (2.19) for n = 2,

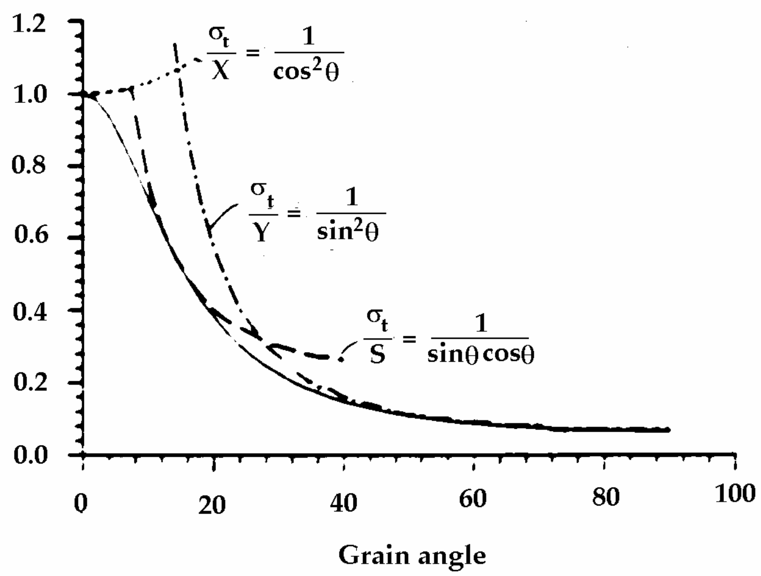
$$\frac{\sigma_t \cos^2\theta}{X} + \frac{\sigma_t \sin^2\theta}{Y} = 1 \tag{3.13}$$

contains the maximum stress condition (or Rankine criterion) of failure for very low and for high angles (see fig. 3.2).

For θ in the neighbourhood of $\theta = 90^0$, eq.(3.13) is about:

$$\frac{\sigma_t \sin^2\theta}{Y} = 1 \tag{3.14}$$

the maximal stress criterion for tension perpendicular to the grain. This also applies down to e.g. 45^0, because 1/X is of lower order with respect to 1/Y and thus the difference of eq.(3.14) with eq.(3.13) is of lower order then. In the same way, for very small values of θ, the ultimate tensile strength criterion in grain direction, eq.(3.15) applies:



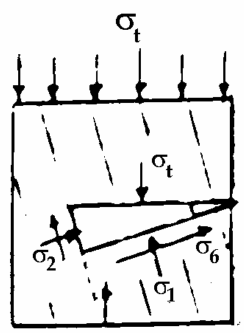


Figure 3.2. - Hankinson and Maximal stress criteria

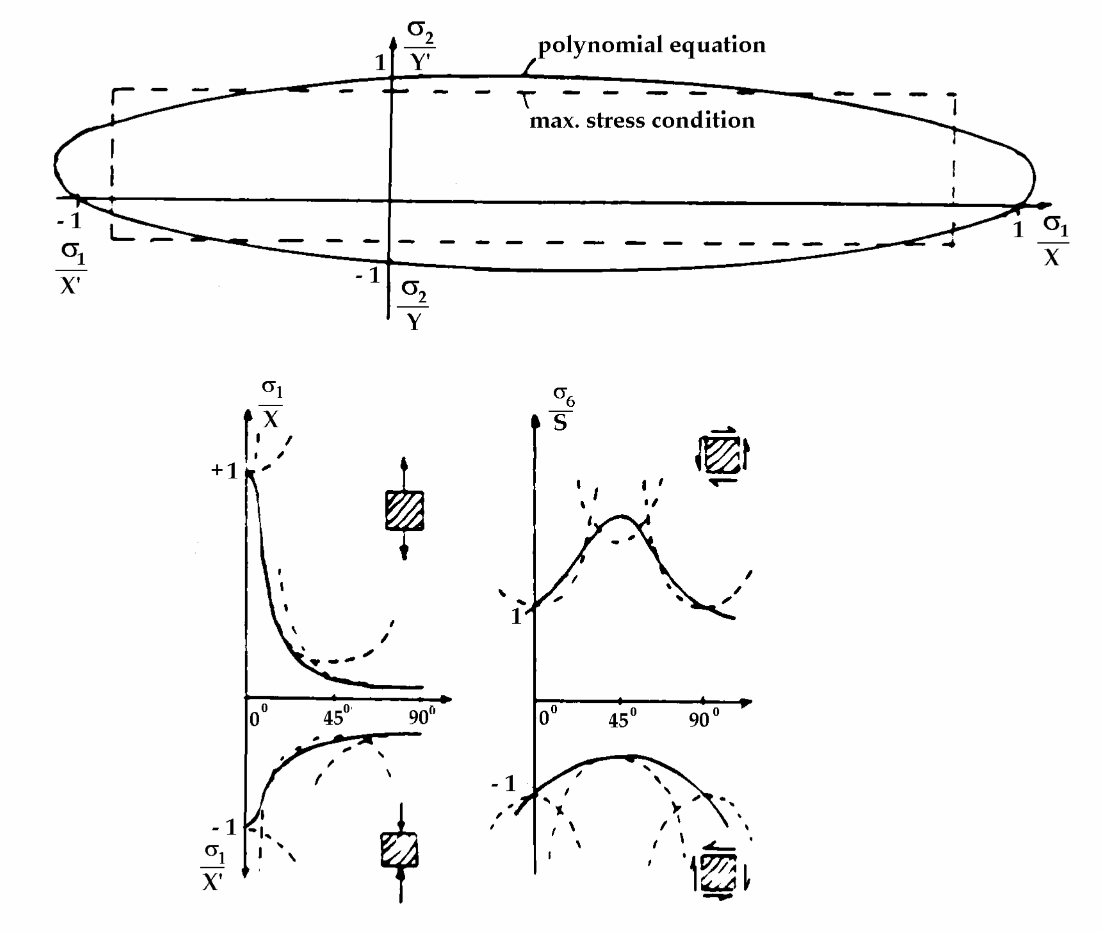


Figure 3.3. - Maximal stress failure conditions.

$$\frac{\sigma_t \cos^2\theta}{X} = 1 \tag{3.15}$$

For values of θ, where the first two terms of eq.(3.13) are equal or:
$\cos\theta/\sqrt{X} = \sin\theta/\sqrt{Y}$,
the deviations of eq.(3.14) and (3.15) from eq.(3.13) are maximal (50%). In the neighbourhood of this value of θ is:
$(\cos\theta/\sqrt{X} - \sin\theta/\sqrt{Y})^2 \approx 0$ or: $\cos^2\theta/X + \sin^2\theta/Y - 2\sin\theta\cos\theta/\sqrt{XY} = 0$
or with eq.(3.13):

$$\frac{\sigma_t \sin\theta\cdot\cos\theta}{\sqrt{XY}/2} = \frac{\sigma_t \sin\theta\cdot\cos\theta}{S} = 1 \tag{3.16}$$

giving the ultimate failure criterion for shear by the fictive shear-strength:
$S = \sqrt{XY}/2$.
It is easy to show that this value of S is the point of contact of the lines eq.(3.16) and eq.(3.13). Although eq.(3.16) fits precisely at this point where $\mathrm{tg}\theta = \sqrt{Y/X}$, the difference of equations (3.14) to (3.16) with eq.(3.13) is to high at their intersects for application (see fig. 3.2). This also follows from figure 3.3 for wood and for other comparable polymers.

3.5. Norris equations

The Norris equations follow from te yield equation, eq.(3.12), when compression and tension strengths are equal: X = X′ and Y = Y′ and thus different equations should be used in each stress quadrant with the strengths X,Y; X′, Y; X, Y′; X′, Y′. When this is done, fig. 3.4 shows that the Norris equations still do not apply.
The success of these equations follows from the uniaxial applications (in the first and third quadrant) when the Hankinson equations apply.
After substitution of X = X′ and Y = Y′, the yield equation, eq.(2.14), can be resolved in factors, like eq.(2.17) as:

$$\left(\frac{\sigma_t \cos^2\theta}{X'}+\frac{\sigma_t \sin^2\theta}{Y'}-1\right)\cdot\left(\frac{\sigma_t \cos^2\theta}{X'}+\frac{\sigma_t \sin^2\theta}{Y'}+1\right)=0$$

showing the Hankinson equations to apply and leading to:

$$\frac{\sigma_t^2 \cos^4\theta}{X'^2}+\frac{\sigma_t^2 \sin^4\theta}{Y'^2}+\frac{\sigma_t^2 \sin\theta^2 \cos^2\theta}{X'Y'}=1 \tag{3.17}$$

This is equal to the Norris-criterion:

$$\frac{\sigma_t^2 \cos^4\theta}{X'^2}+\frac{\sigma_t^2 \sin^4\theta}{Y'^2}-\frac{\sigma_t^2 \sin\theta^2 \cos^2\theta}{X'Y'}+\frac{\sigma_t^2 \sin\theta^2 \cos^2\theta}{S^2}=1 \tag{3.18}$$

when: $1/S^2 \approx 3X'Y'$.
This value of S is measured and can be found in literature (see [1]), showing that the Norris equations are the same as the Hankinson equations for the uniaxial stress case.
For tension (replacing X′ by X and Y′ by Y in eq.(3.18)), it follows in the same way that $S^2 = XY/3$, that may be different from the value for compression, showing that fictive values of S is needed in the other quadrants. Further, the yield criterion eq.(3.12) is an ellipsoid, having a small, (or zero) slope with respect to the σ_1 - axis and thus a negligible F_{12}. The centre of the ellipse in the 1 – 2 – plane is the point: ((X – X′)/2; (Y – Y′)/2). When the part of this ellipse in e.g. the compression – compression quadrant has to be approximated by an ellipse with the centre at the point (0,0), (as applies for the Norris equation), then F_{12} of that ellipse has a pronounced value. In the tension – compression quadrant the apparent F_{12} even has the opposite sign. An improvement of eq.(3.18) thus will be to have a free slope of the ellipses and to use eq.(3.6) in stead as an extended Norris equation.
From eq.(3.17) it follows that:

$$\frac{\sigma_t^2 \cos^4\theta}{X'^2}+\frac{\sigma_t^2 \sin^4\theta}{Y'^2}+\frac{\sigma_t^2 \sin\theta^2 \cos^2\theta}{S'^2}=1 \tag{3.19}$$

when $1/S^2 \approx 2X'Y'$ in eq.(3.17), giving the older empirical Norris equation, that has a zero F_{12} and fits better than the later proposed equation (3.18), but still does not fit in all quadrants (see fig. 3.4) because of the assumed equal compression and tension strengths. Further in all four stress quadrants an other, fictive shear strength has to be used.

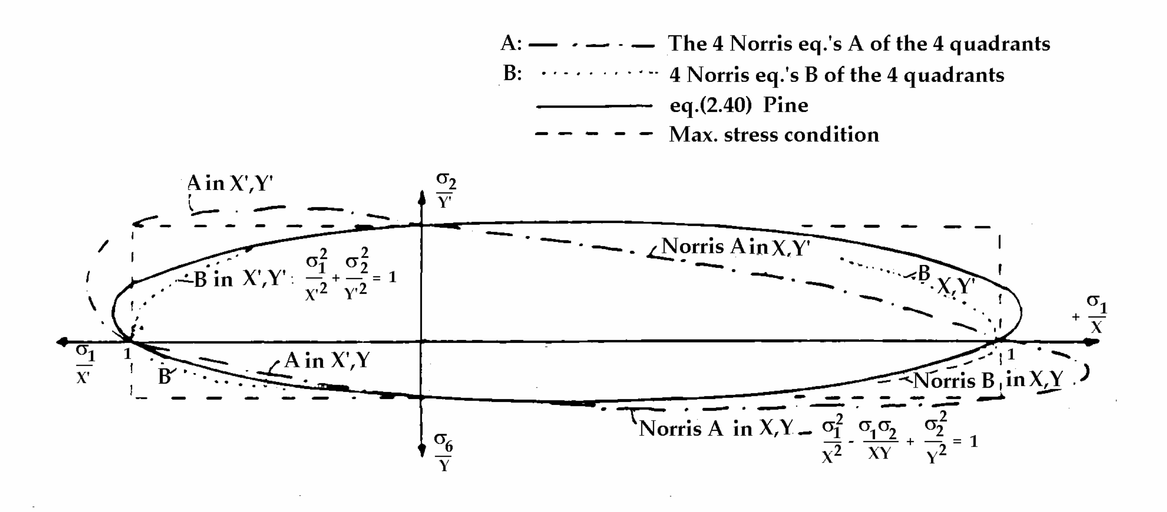


Figure 3.4. - Norris equations for $\sigma_6 = 0$.

It can be concluded that the Norris equations only can be applied for uniaxial stress being equivalent to the Hankinson equations for initial yield.
Because the Norris equations in the general form are not right, they should not be used any more.

3.6. Post yielding behaviour and system hardening

As mentioned before, the loading behaviour up to failure has to be described by deformation kinetics [6]. There are several processes acting in wood. Early local flow or stable micro-crack propagation may occur, while the main part of the material is elastic and different inelastic strain rate equations are necessary depending on the loading type (tension, compression and shear) and loaded material plane. Fig. 2.2 e.g. shows that plastic flow occurs at compression in the radial plane, while in the tangential plane there is flow with hardening. The determining loading behaviour near the ultimate state follows eq.(5.2.8) of [6]. The stress on the specimen is: $\sigma = K_1(\delta/L - \varepsilon_v)$,
where K_1 is the total stiffness of machine and specimen; δ, the crosshead displacement and L the equivalent length of specimen and machine and ε_v the inelastic strain. At a constant crosshead rate $\dot{\delta} = c$, is:

$$\frac{d\sigma}{d(\delta/L)} = K_1 - \frac{LK_1}{c}\dot{\varepsilon}_v = K_1 - K_1(A"+B"\varepsilon_v)\cdot\sinh(\phi\sigma(1-C\varepsilon_v)) \qquad (3.20)$$

As shown in [1], first flow in fig. 2.2 follows: $F'_1\sigma_1 + F'_{11}\sigma_1^2 = 1$. Then hardening occurs. The steepness of the compression hardening curve, after first flow, depends on the orientation of the loaded plane. This means that the value of the hardening constant C in eq.(3.20) depends on this orientation and should be measured. Because there is no yield drop, $A"$ is much higher than $B"\varepsilon_v$. After some mutual equal plastic deformation the stresses are about the same in all

planes, independent of the orientation and the ultimate behaviour becomes quasi isotropic in the transverse direction, indicating yield of the isotropic matrix with further strong hardening by confined dilatation, depending on the type of test.
For tension, the constant C of eq.(3.20) is small and negligible. This means that the ultimate state in a test occurs at the maximal value of σ, thus when:
$d\sigma/d(\delta/L) = 0$, giving:

$$\phi\sigma_u \approx \ln\left(\frac{2}{A''+B''\varepsilon_{v,u}}\right) \tag{3.21}$$

and, A'' and B'' as function of the orientation should be determined. However, as mentioned, the measured values of σ_u can be given as a tensor polynomial for tension perpendicular to the grain in this case, that can be regarded as a polynomial expansion of eq.(3.21).
Because of the small value of $\varepsilon_{v,u}$ the behaviour also can be regarded as quasi elastic up to failure, for tension perpendicular to the grain. This also is applied in practice for tension in grain direction with a reduced modulus of elasticity.
The sometimes quite different reported test results often can be explained by early instability of the test-system for low strength values, or for high values by confined dilatation that may have more or less influence depending on the type of test. This may lead to structure dependent strengths.
Hardening in tension and shear can be explained by stable micro-crack propagation up to long crack lengths. This is caused by strong layers that cannot be passed by the crack and propagation is not in the worst direction but bends off in layer direction causing the hardening effect. Based on this oriented micro-cracks extension in grain direction the parabolic fracture mechanics Wu-interaction equation for mixed I-II mode fracture is derived in [9]. It also appears that this mechanism explains the ultimate failure criterion for wood with small defects. The polynomial third order terms for shear represent this parabolic Wu-equation. But also the third order terms for normal stresses contain this influence because there only is an apparent pure normal stress biaxial loading without shear in wood with defects. This is discussed before (and in [7]). The equations for timber with defects are in principle derivable from the clear wood equations by analyzing the stresses around knots, cracks etc. Descriptive, by the polynomial approach, it is also possible to regard the many possible complicated stress states leading to failure in timber with defects, when loaded in a main direction as the strength by this mean stress in that direction. In that case compression perpendicular to the grain has a strong influence on the axial strengths because without this compression shear with tension perpendicular would be determining and now splitting e.g. around knots and crack propagation (effective shear) is prevented.
Because by grain- and stress- deviation there will always be shear involved in failure and the combined shear strength is determining. The influence of shear with normal force is present in the third order terms for the apparent loading case of the normal stresses only.

As discussed before the hardening is not always present in all tests and a lower bound should be used where also F_{12} can be neglected. Thus for plane stress is:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \tag{3.22}$$

in all cases, what is more easy to use than the not valid Norris criteria.
In general eq.(3.23) applies for the 3-axial stress state, as also is discussed in [1]:

$$\sigma_1\left(\frac{1}{X}-\frac{1}{X'}\right)+\left(\sigma_2+\sigma_3\right)\cdot\left(\frac{1}{Y}-\frac{1}{Y'}\right)+\frac{\sigma_1^2}{XX'}+2F_{12}\left(\sigma_1\sigma_2+\sigma_1\sigma_3\right)+\frac{\sigma_2^2+\sigma_3^2+2\sigma_4^2}{YY'}+$$
$$+\frac{\sigma_5^2+\sigma_6^2}{S^2}=1 \tag{3.23}$$

In this equation σ_4 is the rolling shear and, σ_2 and σ_3 are the normal stresses in the tangential and radial planes. In this equation too, $F_{12} = 0$ can be assumed.
It thus can be concluded that the critical distortional energy criterion, reduced when $F_{12} = 0$, to the critical strain energy criterion, also can be used as a lower bound of the ultimate failure condition.

4. CONCLUSIONS

- The tensor polynomial failure criterion can be regarded as a polynomial expansion of the real failure criterion.
- At initial yield, the minimum energy principle leads to the critical distortional energy principle. It is shown here and in [1], that the general second order polynomial represents this principle for an orthotropic material. This criterion thus is a safe lower bound for failure.
- In transverse direction, a second order polynomial eq.(2.40) is sufficient to describe the strength. For compression perpendicular to the grain, the strength can be defined as the ultimate stress after flow and some amount of strain hardening causing a directional independent, quasi isotropic, behaviour. This also applies for tension perpendicular to the grain because the only local higher tensile strength of the radial plane will not be determining in practice.
- For the longitudinal strength, also a second order polynomial (with $F_{12} = 0$), eq.(2.40), applies as yield criterion. When early failure instability occurs in the test, at initial crack extension, as for instance in the oblique-grain tension test, or for shear with compression in the "Schereisen" test, there are no higher order terms, also not in the radial plane. Higher order terms thus are due to hardening effects depending on the type of test that may provide confined dilatation or stable or unstable crack propagation after initial yield.
- The initial yield equation for uniaxial loading can be resolved into factors containing the Hankinson equation for tension and compression for n = 2. Thus when the Hankinson parameter n in eq.(2.19) is n = 2, in tension and in compression, all

higher order terms are zero. This may apply for clear wood, depending on the type of test. It also is probable that this is a general property for timber [11].
- The yield equation for uniaxial loading, containing higher order terms, can be resolved in factors of the extended Hankinson equations, eq.(2.36) for tension and compression when n in eq.(2.19) is different from n = 2.
- For wood, at least in the radial plane, after hardening in a stable test, the combined longitudinal normal stress - shear strength depends on the third order coupling term F_{266}, giving the parabolic Mohr- or Wu- equation of fracture. This is theoretically explained in [9] by micro-crack propagation in grain direction. This increase of the shear strength is an equivalent hardening effect due to crack arrest in the worst direction by strong layers, causing failure only to be possible by longitudinal crack propagation. It is shown that the increase of the shear strength, by compression perpendicular to the shear plane, is not due to Coulomb friction, being small for wood.
- Because of the grain deviations from the regarded main directions, there always is combined shear-normal stress loading in the real material planes where eq.(2.32) applies and F_{112} probably is an apparent value caused by F_{266} of the real inclined material planes.
- Therefore, for wood in longitudinal compression in the radial plane this micro-crack failure mechanism is determining, giving high values of F_{266} and F_{112}, close to their bounds of $c \approx 0.8$ to 0.9.
- The same as found for F_{266} as function of σ_2, is to be expected for F_{166} as function of σ_1. This is given in fig. 2.7.
- For wood in longitudinal tension, F_{12}, F_{112} and F_{122} are zero and only F_{166} and F_{266} remain in the radial plane as higher order terms, showing an other type of failure than for longitudinal compression.
For longitudinal compression, at $\sigma_6 = 0$, equivalent hardening by crack arrest, (high F_{112}) as well hardening by confined dilatation (showing a negative F_{122} and F_{12}) may occur. This last type of hardening occurs in the torsion tube test, because the negative F_{122} and F_{12} predict the compression peak of fig. 2.4 in the oblique grain test, that does not occur by the lack or hardening in the oblique grain test. This also will be so for structural elements and the lower bound criterion with only F_{166} and F_{266} (and zero F_{12}, F_{112} and F_{122}) is probably more reliable for longitudinal compression failure in the radial plane. In the tangential plane also F_{166} and F_{266} are zero, making the second order criterion determining.
- In general thus eq.(3.23) applies for the 3-axial stress state, as is discussed in [1]:

$$\sigma_1\left(\frac{1}{X}-\frac{1}{X'}\right)+\left(\sigma_2+\sigma_3\right)\cdot\left(\frac{1}{Y}-\frac{1}{Y'}\right)+\frac{\sigma_1^2}{XX'}+\frac{\sigma_2^2+\sigma_3^2+2\sigma_4^2}{YY'}+\frac{\sigma_5^2+\sigma_6^2}{S^2}=1$$

where σ_4 is the rolling shear and σ_2 and σ_3 are the normal stresses in the tangential and radial planes and where it is assumed that $F_{12} = 0$ as applies for longitudinal tension.
- Equations (3.38) and (2.39) can be used for analyzing test data. Because it is questionable that the hardening by confined dilatation or crack arrest may occur in all circumstances, because it depends on the type of test, the hardening contained by the third order terms should be omitted for a general application.
- Therefore the second order polynomial, eq.(3.22) or eq.(2.40), for plane stress:

$$\frac{\sigma_6^2}{S^2}+\frac{\sigma_1}{X}-\frac{\sigma_1}{X'}+\frac{\sigma_1^2}{XX'}+\frac{\sigma_2}{Y}-\frac{\sigma_2}{Y'}+\frac{\sigma_2^2}{YY'}=1$$

should be used for initial yield and for ultimate failure for the Codes and as initial yield equation, it applies for the 5th percentile of the strength as well.
- The loading curve up to yield and failure should be described by deformation kinetics [6].
- It is shown that the quadratic form of the tensor-polynomial yield criterion represents the critical distortional energy principle for initial yield and the Hankinson and Norris equations may represent special forms of this principle.
- Only this derived extension of the von Mises criterion contains the, for orthotropic materials, necessary independent value of the interaction constant as F_{12} and accounts for different tension- and compression strengths and is able to give the strength in any direction in the strength tensor form.
- The ultimate stress principle for failure, eq.(3.14), (3.15) and (3.16), does not apply for the general loading case and only applies locally and approximately for only uniaxial loading. These equations also are predicted by the fracture mechanics singularity method [9], showing thus that this method, that always is applied in fracture mechanics for all materials, is not right and should not be used.
- The Norris equations are not generally valid and are only for uniaxial loading identical to the Hankinson equation with n = 2, when the right (mostly) fictive shear-strength is used. This equation thus should not be used any more.
- There thus is no reason to not apply this exact general criterion, also for the future Codes, for all cases of combined stresses. Only this criterion gives the possibility of a definition of the off-axis strength of anisotropic materials.

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