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TENSION PERPENDICULAR TO THE GRAIN AT NOTCHES AND JOINTS

by

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Summary

Constructive details as notches may cause high tension perpendicular to the grain and should not be applied. However rules are necessary if alternative solutions are not possible and design rules are proposed for the Dutch Code TGB-1990 as a better alternative than the Eurocode rules 5.1.7.1 and 5.3.1.
Although the background of the design rules of the American Code for notched beams is not known it is possible to derive these rules and it is shown that they are only applicable for narrow span, high beams.
Design rules are derived using the simple fracture mechanics approach of [1]. Except for splitting along the grain the method is also used for crack propagation perpendicular to the grain. The same is applied for joints at the lower edge of a beam leading to an equivalent instable crack length. Simple design rules appear to be possible based on a lowest upper bound of the strength leading to the equations of 5.1 for notched beams and of 5.2 for excentric joints.

1. Introduction

Structural details like notches, causing high tensile peak stresses perpendicular to the grain, should be regarded as building faults and beside the design rules for exceptional cases the codes should provide solutions (as given in [3]) eliminating the influence of these peak-stresses.
The stress situation for connections at the tensional side of a beam may be simular to that of notched beams and often the same design rules are given for these joints. In [1] a method is given to explain the behaviour of notches and probably this can be extended, as proposed here, to explain the behaviour of splitting joints as well. The results of [1] show a good agreement between theory and tests except for short beams showing that an other mechanism is determining in this case. Probably this is the result of crack propagation perpendicular to the grain at weak spots and a first lower bound estimation is given here.

2. Explanation of the U.S. design rules for notched beams and for connections at the tensional side of a beam

Although the background of the design rules for beams with notches at the ends in the different codes cannot be found in literature and is not known any more a derivation of these rules is possible and will be given below. For connections at the lower part of the beam eq.(5) or (6) are used and it will be shown that this only ap-

plies for short beams.
A crack may propagate from the corner of a notch at the support along the grain until the loading point in the field is reaches in a three- or four— point bending test. The lower bound of the strength is thus determined by the strength of the remaining beam, after the crack propagation, with a height h_e.
If a beam according to fig. 1 may fail, at the same time, by bending or by shear the shear force V_d will be:

$$V_d = M_d/L = \frac{f_{o,d} \cdot bh^2}{6L} \tag{1}$$

and in the same time:

$$V_d = \frac{2}{3} f_{v,d} \cdot bh \tag{2}$$

Thus from (1) and (2):

$$\frac{h}{L} = \frac{4f_{v,d}}{f_{o,d}} \tag{3}$$

When this beam is notched at the ends (fig. 2) the strength of the remaining beam after crack propagation is:

$$V_d = M_d/L = \frac{f_{o,d} \cdot bh_e^2}{6L} \tag{4}$$

or with eq.(3):

$$V_d = \frac{2}{3} f_{v,d} \cdot bh_e \cdot \frac{h_e}{h} \tag{5}$$

This equation for the strength reduction by a notch is the Code equation of the U.S.A and was proposed for the first draft of the Eurocode.
When bending failure is not determining for the unnotched beam is: $h/L > 4f_{v,d}/f_{o,d}$ and is V_d higher than the value according to eq.(5) with a maximum of:

$$V_d = \frac{2}{3} f_{v,d} \cdot bh_e \tag{6}$$

This occurs when: $h_e/L \geq 4f_{v,d}/f_{o,d}$ and thus also shear failure of the remaining beam is determining for the strength.
When only bending failure is determining ($h/L < 4f_{v,d}/f_{o,d}$), V_d is lower than according to eq.(5) and is for the remaining beam:

$$V_d = \frac{f_{o,d} \cdot bh_e^2}{6L} = \frac{2}{3} f_{v,d} \cdot bh_e \cdot \frac{f_{o,d}}{4f_{v,d}} \cdot \frac{h_e}{L} \tag{7}$$

This, also measured, lower value with respect to eq.(5) should be used for longer beams if there is not accounted for a strength perpendicular to the grain. The measurements show that eq.(7) applies for the failure mode of splitting along the grain and is safe for small specimens (when the splitting strength is high).

It can be concluded that the design of notched beams can be based on the strength of the remaining beam (that would remain after crack propagation) when cracking of the beam need not to be regarded as a limit state of the utility of the beam.
If the splitting is regarded as a limit state of the beam the higher strength of the small beams and lower "strength" of the large beams have to be estimated by fracture mechanics as given in 3.

3. Explanation of the strength of notched beams and derivation of design rules

In the following the simple fracture mechanics approach of [1] is followed, with a slightly different starting point, to estimate the bound for splitting and the simple beam theory is used for the determination of the deformation δ by crack propagation. In the neighbourhood of the cracktip the stresses deviate from the beam theory according to an internal equilibrium system and it can be assumed that the dissipation by this system will not cause an increase of δ and is the same for all beams (Every flat crack has the same stress gradient). So an apparent value of the fracture energy is determined. This is not a disadvantage here because accounting for these effects would also provide an apparent value in this model because of the mixed mode crack propagation.
The potential energy of the symmetrical half of the beam according to fig. 2 is:
$W = V\delta/2$. When V is constant the increase of the crack length with Δx will increase the deflection with $\Delta\delta$. When the lost of the potential energy ΔW becomes equal to the energy of crack formation, crack propagation occurs. The energy of crack formation is: $G_c b\Delta x = G_c bh\Delta\beta$, where G_c is the crack formation energy per unit crack area. Thus crack propagation occurs at $V = V_f$ when:
$\Delta W = V\Delta\delta/2 = V^2\Delta(\delta/V)/2 = G_c bh\Delta\beta$, or when:

$$V_f = \sqrt{\frac{2G_c bh}{\frac{\partial\delta/V}{\partial\beta}}} \qquad (8)$$

The change of δ by the increase of shear deformation is with $h_e = \alpha h$:

$$\delta_v = \frac{2}{G}\left(\frac{\beta h}{b\alpha h} - \frac{\beta h}{bh}\right)\cdot V \qquad (9)$$

The change of δ by the increase of the deflection is:

$$\delta_m = \frac{V(\beta h)^3}{3Eb(\alpha h)^3/12} - \frac{V(\beta h)^3}{3Ebh^3/12} = \frac{4V\beta^3}{Eb}\cdot\left(\frac{1}{\alpha^3} - 1\right) \qquad (10)$$

Thus:

$$\frac{\partial\delta/V}{\partial\beta} = \frac{2}{Gb}\cdot\left(\frac{1}{\alpha} - 1\right) + \frac{12\beta^2}{Eb}\cdot\left(\frac{1}{\alpha^3} - 1\right) \qquad (11)$$

The critical value of V thus is according to eq.(8):

$$V_f = \sqrt{\frac{G_c h b^2}{\frac{1}{G}\left(\frac{1}{\alpha} - 1\right) + \left(\frac{1}{\alpha^3} - 1\right)\cdot\frac{6\beta^2}{E}}} \tag{12}$$

or:

$$\frac{V_f}{b\alpha h} = \frac{\alpha\sqrt{GG_c/h}}{\sqrt{\alpha^3 - \alpha^4 + 6\beta^2(\alpha - \alpha^4)G/E}} \tag{13}$$

For small values of β eq.(13) becomes:

$$\frac{V_f}{b\alpha h} = \frac{\alpha\sqrt{GG_c/h}}{\sqrt{\alpha^3 - \alpha^4}} \tag{14}$$

For high values of β, $\beta = c\eta$ with $\eta = L/h$, eq.(13) becomes with $E/G = 30$:

$$\frac{V_f}{b\alpha h} = \frac{\alpha\sqrt{GG_c/h}}{\beta\sqrt{0{,}2\cdot(\alpha - \alpha^4)}} = \frac{\alpha\sqrt{GG_c/h}}{c\eta\sqrt{0{,}2\cdot(\alpha - \alpha^4)}} \left(= \frac{\alpha K_1}{c\eta\sqrt{6h(\alpha - \alpha^4)}}\right) \tag{15}$$

where K_1 is the stress intensity factor.
Because $\sqrt{\alpha - \alpha^4}$ doesn't change much with the usual values of α this equation is comparable with eq.(7) for the lower bound of the strength and depending on the value of c, V_f will be higher or lower than V according to eq.(7) and crack propagation will be instable respectively stable.
An example of measured high values of β can be found in the investigation of Murphy, mentioned in [1], done on a notch starting at $\beta = 2.5$ and proceeding to $\beta = 5.5$ ($\eta = 10$, or $c = 0.55$). Further also beams are tested with a cut at a distance $\beta = 2.5$ ($\eta = 10$, or $c = 0.25$). Because of the high value of β eq.(15) aproximately applies and the measurements show a mean value of $\sqrt{GG_c} = 8.9$ N/mm^{1,5}. For all specimens was: $\alpha = 0.7$; $\eta = 10$; $b = 79$ mm. The other data are given in table 1.

Table 1. Strength of clear laminated Douglas fir with notches in the tensile zone in MPa (Murphy)

h	β	num-	$V/\alpha bh$	
mm		ber	tests	eq.(15)
305	2.5	2	0.46	0.47
305	5.5	2	0.24	0.22
457	2.5	2	0.38	0.38
457	5.5	1	0.16	0.17

From the table it follows that for high values of η, the strength, also at high values

of β, is only determined by eq.(15) or only by horizontal crack propagation. An estimation of the conditions for the bend off of the crack can be made by determining the crack propagation in vertical direction.
The energy of crack formation in y-direction is:

$$G_m b\Delta y = G_m bh\Delta\alpha = V\Delta\delta/2 = V^2\Delta(\delta/V)/2.$$

Thus crack propagation occurs when:

$$V_m = \sqrt{\frac{2G_m bh}{\frac{\partial\delta/V}{\partial\alpha}}} \qquad (16)$$

If it is assumed that vertical crack propagation is accompanied with horizontal crack propagation over a long distance than equations (9) and (10) apply for δ and is, with $\beta h = L$ (as lower bound):

$$-\frac{\partial\delta/V}{\partial\alpha} = \frac{2\eta}{Gb}\cdot\left(-\frac{1}{\alpha^2}\right) - \frac{12\eta^3}{Eb}\cdot\frac{1}{\alpha^4} \qquad (17)$$

or, according to eq.(16):

$$\frac{V_m}{b\alpha h} = \frac{\alpha\sqrt{GG_m/h}}{\sqrt{\alpha^2\eta + 6\eta^3 G/E}} = \frac{\alpha\sqrt{GG_m/h}}{\eta\sqrt{\alpha^2/\eta + 6\eta\ G/E}} \qquad (18)$$

Application of this equation on the data of table 1 for $\beta = 5.5$ and $E/G = 30$ shows that $\sqrt{GG_m}$ has to be smaller than 70 to 85 $N/mm^{1.5}$ (if this mechanism was determining at the same time with horizontal splitting).

In order to get simplifications for the Code it can be seen that the variation of the denominator of eq.(13) is not much at smaller values of β and the usual values of α so that as a first estimate:

$$\frac{V_f}{\alpha bh} = \alpha f_v \sqrt{\frac{h_v}{h}} \qquad (19)$$

Eq.(18) also doesn't vary much with the denominator under the square root sign for the values of η wherefore this equation is determining so that as a first estimate:

$$\frac{V_m}{\alpha bh} = \frac{\alpha}{\eta} f_{mo}\sqrt{\frac{h_m}{h}} = \alpha f_v \sqrt{\frac{h_v}{h}}\cdot\frac{\eta_0}{\eta} \qquad (20)$$

where h_v, h_m and η_0 are constants. The last equation is determining when $\eta \le \eta_0$ because crack propagation occurs at V_f according to eq.(19) as if $\eta = \eta_0$ in eq.(20) and hardening can occur after crack formation when $\eta \le \eta_0$.

An example whereby the shear strength is determining for the un-notched beam, $V = (2/3)bhf_v$ is measured by Kollmann and given in [1]. Because only the ratio of the strength of the notched beam with respect to the strength of the un-notched

beam is published the test results are calculated by assuming $f_v = 10$ N/mm^2 for "Red tulip oak". For this case of small η the strength of the remaining beam is high and there will be no instant failure after the occurence of the horizontal crack.
In table 2 the crack formation energies are calculated according to the different equations.

Table 2. Strength of notched beams, Red tulip oak, Kollmann.

h	α	β	η/α	b	n	$\frac{V}{b\alpha h}$	var. coef.	$\sqrt{GG_f}$	$\sqrt{GG_m}$	$f_v\sqrt{h_v}$	$f_m\sqrt{h_m}$
								tests		approximations	
mm				mm		N/mm^2	%		N/mm$^{1.5}$		
100	.875	~ 0.3	2.0	50	1	5.56	-	18.4	101	63.5	111
	.75		2.4		2	3.47	-	15.0	68.3	46.3	83.3
	.625		2.9		1	2.77	-	13.4	60.6	44.3	80.3
	.5		3.6		2	2.53	-	12.7	64.3	50.6	91.1
	.25		7.2		1	~1.9	-	8.2	85.9	76	137

Splitting is possibly determining for $\alpha = 0.25$ met $\sqrt{GG_f} = 8.2$, comparable with Douglas Fir. For higher values of α vertical crack prpagation, or bending failure of the remaining beam, is determining with:
$\sqrt{GG_m} = 64{,}4$ N/mm$^{1.5}$.
Possibly there is a more favourable mechanism at $\alpha = 0.875$ (lower splitting stresses). For this case (and for $\alpha = 0.25$) eq.(5) is satisfied or: $V_d/b\alpha h = 2/3 \cdot \alpha f_{v,d}$.
In the following tables the crack formation energy is determined for the other cases of [1].

Table 3. Strength of notched beams depending on the height of the beam

h	α	β	η/α	b	n	$\frac{V}{b\alpha h}$	var. coef.	$\sqrt{GG_f}$	$\sqrt{GG_m}$	$f_v\sqrt{h_v}$	$f_m\sqrt{h_m}$
								tests		approximations	
mm				mm		N/mm^2	%		N/mm$^{1.5}$		
5 min. test, m.c. 14.9 %, ρ = 467 kg/m^3, Gustafsson, Pinus sylvestris L.											
12	.75	.5	3.6	44	7	3.32	16	5.5	35.8	15.3	41.4
48						2.75	10	9.1	59.3	25.4	68.6
196						1.3	25	8.7	56.7	24.3	65.5

Table 4. Strength of notched beams depending on the height of the beam (continuation of table 3)

h	α	β	η/α	b	n	$\frac{V}{b\alpha h}$	var. coef.	$\sqrt{GG_f}$	$\sqrt{GG_m}$	$f_v\sqrt{h_v}$	$f_m\sqrt{h_m}$
								tests		approximations	
mm				mm		N/mm^2	%		N/mm$^{1.5}$		
m.c. 12 %, Carlson, Shahabi, Sunding, Pinus sylvestris											
50	.5	.5	10	45	2	2.0	-	8.2	145	28.3	141
100			5			1.46	-	8.5	56.5	29.2	73
200			2.5			1.18	-	9.7	28.0	33.4	42
m.c. 18 %, Gustafsson, Pinus sylvestris											
45	.5	.5	6.7	45	6	1.72	9	6.7	66.7	23.1	77.3
195			6.2			0.93	17	7.5	67.4	26.0	80.5
m.c. 18 %, Gustafsson, Pinus sylvestris											
45	.5	.5	6.7	45	4	1.92	9	7.5	74.5	25.7	86.3
195			6.2			0.96	4	7.8	69.6	26.8	83.1
Eucalyptus, Leicester											
9.5	.5	1.92	9.3	38	4	3.9	-	14.9	110.9	24.0	111.8
19					4	3.08	-	16.7	123.8	26.8	124.9
37					≥ 2	1.9	-	14.3	106.6	23.1	107.5
58					4	1.77	-	16.7	124.3	27.0	125.4
154					4	1.07	-	16.5	122.5	26.6	123.5

For very small specimens (h = 12 mm) the crack length has to be adjusted (see [1]). It follows from table 3 that for Pinus sylvestris: $\sqrt{GG_f}$ = 8.1 and: $f_v\sqrt{h_v}$ = 25.6 N/mm$^{1.5}$. For Eucalyptus this is resp. 15.8 en 25.9 N/mm$^{1.5}$. The two times higher crack formation energy of Eucalyptus is not shown in the approximation value. Thus the approximation only can be used for the usual applied small values of β.

In the next table the measurements are given for Spruce wherefore also vertical crack propagation is determining. Because for all softwoods the same crack formation energy is measured the values of table 5 can be used in all cases.

Table 5. Strength of notched beams, Spruce, Moher en Mistler.

h	α	β	η/α	b	n	$\frac{V}{b\alpha h}$	var. coef.	$\sqrt{GG_f}$	$\sqrt{GG_m}$	$f_v\sqrt{h_v}$	$f_m\sqrt{h_m}$
								tests		approximations	
mm				mm		N/mm^2	%		$N/mm^{1.5}$		
testing time ore than 1 min., clear, m.c. 11 %, ρ = 510 kg/m^3											
120	.917	.25	3.4	32	6	2.36	11	7.3	84.1	28.2	87.9
	.833		3.8		27	1.93	15	8.1	73.7	25.4	80.3
	.75		4.2		43	1.68	19	8.2	69.5	24.5	77.3
	.667		4.7		14	1.52	18	8.1	69.1	25.0	78.3
	.583		5.4		10	1.5	18	8.4	76.3	28.2	88.7
	.5		6.3		49	1.59	18	9.1	92.4	34.8	109.7
	.333		9.5		10	1.48	16	8.2	125.1	48.7	154.0
gluelam. Spruce. ρ = 470 kg/m^3.											
600	.917	.417	2.2	100	5	2.00	13	14.3	96.8	53.4	107.8
	.833		2.4		4	1.61	28	15.6	81.8	47.3	94.6
	.75		2.7		4	0.88	12	10.0	47.4	28.7	58.2
	.667		3.0		4	0.86	16	10.7	49.8	31.6	63.2
	.5		4.0		4	0.75	7	10.2	53.2	36.7	73.5

It follows from the table for Spruce: $\sqrt{GG_f}$ = 8.4 and: $f_v\sqrt{h_v}$ = 28.5 $N/mm^{1.5}$ ($\alpha \geq$ 0.5). The representative value is about: (1 - 1,64·0.2) = 0.67 times as high.

For gluelam there is possibly a more favourable crack mechanism at $\alpha \leq$ 0.833 (as also follows from the two times higher coefficient of variation at this boundary value of 0,83). A safe value for: $\sqrt{GG_m}$ = 50 and $f_m\sqrt{h_m}$ = 65 $N/mm^{1.5}$ so that: η_0 = = 65/28.5 = 2,3.

The Australian code also shows for timber an increase in strength at $\alpha \geq$ 0,9.

4. Explanation of the strength of connections at the lower boundary of a beam and derivation of design rules

For a connection at the middle of a beam the following applies after splitting (see fig. 3). The part above the crack (stifness I_2) only carries a moment (M_2) and the part below the crack (stiffness I_1) carries a moment (M_1) and the shearforce (V).

The rotation φ at the end of half the crack length $\lambda = \beta h$ then is:

$$\varphi = \frac{M_2\lambda}{EI_2} = \frac{M_1\lambda}{EI_1} + \frac{V\lambda^2}{2EI_1} = \frac{(M - M_1)\lambda}{EI_2} \tag{21}$$

with: $M = M_1 + M_2$ being the moment at the end of the crack or:

$$M_1\left(1 + \frac{I_2}{I_1}\right) = M - \frac{V\lambda I_2}{2I_1} \text{ and } \varphi = \frac{\bar{M}\lambda}{E(I_1 + I_2)} \tag{22}$$

$\bar{M}$ is the mean moment over the length λ ($\bar{M} = M + V\lambda/2$).
According to fig. 3 this is the mean moment over the length βh: $\bar{M} = VL(1 - \beta h/2L)$ and the relative deflection of the support follows from: $\delta = \delta_v + \varphi(L - \beta h/2)$ so that the increase of the deflection by splitting is:

$$\delta = \frac{2}{G}\left(\frac{\beta h}{b\alpha h} - \frac{\beta h}{bh}\right)\cdot V + VL\left(1 - \frac{\beta h}{2L}\right)\frac{\beta h}{E}\left(L - \frac{\beta h}{2}\right)\cdot\left(\frac{12}{(\alpha h)^3 + (1 - \alpha)^3 h^3} - \frac{12}{h^3}\right) =$$

$$= \frac{2}{G}\left(\frac{\beta h}{b\alpha h} - \frac{\beta h}{bh}\right)\cdot V + \frac{12VL^2}{Eh^2}\cdot\left(1 - \frac{\beta h}{2L}\right)^2\cdot\beta\cdot\frac{3\alpha(1 - \alpha)}{1 - 3\alpha(1 - \alpha)} \tag{23}$$

so that:

$$\frac{\partial\delta/V}{\partial\beta} = \frac{2}{Gb}\cdot\left(\frac{1}{\alpha} - 1\right) + \frac{12\eta^2}{Eb}\cdot\frac{3\alpha(1 - \alpha)}{1 - 3\alpha(1 - \alpha)}\cdot\left(1 - \frac{\beta h}{2L}\right)\cdot\left(1 - \frac{3\beta h}{2L}\right) \tag{24}$$

and eq.(8) becomes for small values of β of the initial crack with $E/G = 30$:

$$\frac{V_f}{b\alpha h} = \frac{\sqrt{GG_c/h}}{\sqrt{\alpha - \alpha^2 + 0.6\eta^2(\alpha^3 - \alpha^4)\cdot(1 - 2\beta/\eta)/(1 - 3\alpha + 3\alpha^2)}} \tag{25}$$

The second term in the denominator has a small influence for small values of α and for increasing (stable) crack increase (= increase of β) the equation is, for $\beta h \approx L/2$, equal to eq.(14) giving the same equation as for notched beams with short notches at the ends. Eq.(14) is here an upper limit because there is no exchange of energies of shear- and split- deformation. So it is to be expected that the measured failure data will not be far below the values according eq.(14) and $\beta \approx \eta/2$ can be used as approximation.
In the measuring range of [2] eq.(25) can be written for $\alpha \approx 0.15$ to 0.55:

$$\frac{V_f}{b\alpha h} \approx \frac{\sqrt{GG_c/h}}{\sqrt{(\alpha - \alpha^2)\cdot(1 + 5.4\eta(\eta - 2\beta)\alpha^3)}} \tag{26}$$

With this equation a first estimate of β can be made for sufficient small values of α and sufficient high values of η. For higher values of α ($\alpha \geq 0.7$) splitting will not occur, at least not in the measuring range of [2] where is measured on relatively short beams. For small values of η the strength of the remaining beam may be determining so that the horizontal crack propagation remains stable until failure of this beam (the start of cracking in vertical direction). If, by the higher loading, only I_1 changes by crack formation perpendicular to the grain it follows from eq.(23):

$$\frac{\partial \delta/V}{-\partial \alpha} = - \frac{2}{Gb}\cdot\left(\frac{\beta}{\alpha^2}\right) - \frac{12\beta\eta^2}{Eb}\cdot\left(1 - \frac{\beta}{2\eta}\right)^2\cdot\frac{3\cdot\alpha^2}{\left(\alpha^3 + (1 - \alpha_0)^3\right)^2} \tag{27}$$

or according to eq.(16) is for $\alpha = \alpha_0$:

$$\frac{V_m}{b\alpha h} = \frac{\sqrt{GG_m/h}}{\sqrt{\beta + 18\beta\alpha^4(\eta - \beta/2)^2 G/(E\cdot(\alpha^3 + (1 - \alpha)^3)^2)}} \tag{28}$$

For $\beta = \eta$, as lower bound, this becomes:

$$\frac{V_m}{b\alpha h} = \frac{\sqrt{GG_m/h}}{\sqrt{\eta + 4{,}5\,\alpha^4\eta^3 G/(E\cdot(\alpha^3 + (1 - \alpha)^3)^2)}} \tag{29}$$

In the measuring range of [2], eq.(29) can be written for $\alpha \approx 0{,}15$ to $0{,}55$:

$$\frac{V_m}{b\alpha h} \approx \frac{\sqrt{GG_m/h}}{\sqrt{\eta + 7{,}9\cdot\eta^3\alpha^6}} \quad \left(\geq \frac{\sqrt{GG_m/h}}{\sqrt{\beta_m + 31\cdot\beta_m\alpha^6(\eta - \beta_m/2)^2}}\right) \tag{30}$$

The second term in the denominator has a small influence for small values of α and eq.(30) is therefore about comparable with eq.(18) for notches (for small η).
For higher values of η (not measured in [2]) for instance at $\eta \geq 5$ the strength is high according to the American Code and independent of η. Then the lower bound $\beta = \eta$ according to eq.(29) doesn't occur but $\beta = \beta_{max}$ = constant en $V_m/b\alpha h \approx \approx \sqrt{GG_m/h\beta_m} \approx$ constant $\geq f_v$ in the measuring range of the American Code as given in eq.(30).
Eq.(31) (or (14)), (29) and (25) are tested in the following tables. In the measuring range of small values of η both equatons (32) and (29) may be determining. According to eq.(14) is:

$$\sqrt{GG_c} = \frac{V_f}{\alpha h}\cdot\frac{\sqrt{h}}{b}\cdot\sqrt{\alpha\cdot(1 - \alpha)} \tag{31}$$

By multiplication with: $\sqrt{1 + 5{,}4\alpha^3\eta(\eta - 2\beta)}$ according to eq.(26) a first estimate is possible of β, in the un-cracked stage, for higher values of η.
For simplicity mean values of β are regarded in the table. By this $\sqrt{GG'_c}$ is too high for higher values of α and too small for small α in series b where β has influence. However the differences are not much and will have no influence on the mean value of $\sqrt{GG'_c}$.

Table 6. Strength of the connection, Möhler en Lautenschläger

h mm	b mm	type	d mm	numb. per row	numb. of rows	numb. of tests	αh mm	$\frac{V}{b\alpha h}$ N/mm^2	$\sqrt{GG_c}$ eq.(14) N/mm$^{1.5}$	2β	$\sqrt{GG'_c}$ eq.(25)	η	$\sqrt{GG_m}$ eq.(29)
Assumption L/h = 2.5 (L is not given in [2] and sometimes different from 2.5h)													
pindowels													
180	40	a	8,0	1	1	1	28	2,32	11,3	0	12,1	2,5	49,2
120						3		2,70	12,5		15,0		47,1
nails													
180	40	b/c	3,8	5	1	5	28	3,54	17,2	1,2	17,8	2,37	73,2
						3		3,92	19,1		19,7		81,0
					mean	8		3,68	17,9		18,5		76,1
		b	"	5	1	1	47	2,85	16,8		18,9		59,5
						3		2,54	15,0		16,9		53,0
					mean	4		2,62	15,5		17,4		54,7
				5	1	3	66	2,26	14,6		19,2		50,4
							85	1,97	13,2		20,7		52,4
							104	2,27	15,1		26,5		71,2
180	40	c	"	5	2	1	47	3,59	21,2	> 2.4	21,2		75,0
						3		3,32	19,5		19.5		69,3
					mean	4		3,38	19,9		19,9		70,6
				5	3	3	66	3,57	23,1		23,1		79,6
				5	4	3	85	3,11	20,8		20,8		82,7
				5	5	3	104	3,34	22,2		22,2		104.7
120	40	d		2	1	3	28	4,25	19,7	0	22,8	2,18	69,1
								3,60	16,7		19,5	2,26	59,6
								3,04	14,1		16,6	2,34	51,2
								2,87	13,3		15,8	2,42	49,2
		e		1	1	3		3,10	14,4		17,3	2,5	54,0

For types a, d and e in the table where V is carried by 1 or 2 nails (or half a dowel) failure of the connection is probably determining for the strength and splitting is due to a secondary stress concentration after proceeded "plastic" deformation and hardening by failure of the dowel mechanism. So splitting of the wood is not the primary failure mechanism.

In table 7 measurements are given for small values of η. It can be expected that failure of the remaining beam is determining. If the highest value (α = 0.75) is not regarded (because splitting will not occur) the mean value or $\sqrt{GG_m}$ = 53,9 N/mm$^{1.5}$. For each parameter only one test is done except in one case where 2 specimens show values of $\sqrt{GG_m}$ of 46 and 61 N/mm$^{1.5}$. This shows a high variation of this parameter because there is no well defined initial crack length.
The value of $\sqrt{GG_c}$ of 21.8 N/mm$^{1.5}$ according to eq.(14) in table 7, has to be higher than the real value of $\sqrt{GG_c}$ (by hardening after splitting). This means that for the connection type c of table 6 eq.(14) applies or that the critical crack-length $2\beta \approx \eta$ = = 2.37 because then the smallest value of $\sqrt{GG_c} \approx$ 20 N/mm$^{1.5}$ is reached (being smaller). The calculated value of $\sqrt{GG_m}$ is high indicating immediate failure (of the remaining beam) after splitting.

Table 7. Strength of the connection, Möhler en Siebert

h mm	b mm	type	d mm	numb. per row	numb. of rows	numb. of tests	αh mm	$\frac{V}{b\alpha h}$ N/mm^2	$\sqrt{GG_c}$ eq.(14) N/mm$^{1.5}$	η	$\sqrt{GG_m}$ eq.(29)
nails (type n)											
1200	100	n	4,2	10	4	1	300	1,24	18,6	1,42	51,3
							600	0,99	17,1		46,6
pinsowels (type s)											
1200	100	s	16,0	3	2	1	300	1,5	22,5	1,43	62.3
							900	1,06	15,8		53,9
				3	4		300	1,87	28,0		77,7
				3	6		600	1,49	25,9		70,6
				2	2		300	1,08	16,3	1,37	44,0
600	100			3	4	1	300	1,83	22,4	0,94	46,4
						1		2,40	29,4		60.8
					gem.	2		2,12	25,9		53,7
				3	2	1	450	2,0	21,2		52,6
						1		2,22	23,6		58,3
						1		3,44	25,9	1,4	(121,5)
						1	150	2,32	24,6		67,4
						1		2,08	22,1	0,44	34,0
						1	300	1,25	15,3	1,4	41,2
				2	2	1	150	1,87	19.8	0,83	41,8

Table 8. Strength of the connection. Ehlbeck en Görlacher

h mm	b mm	type	d mm	numb. per row	numb. of rows	numb. of tests	αh mm	$\frac{V}{b\alpha h}$ N/mm^2	$\sqrt{GG_c}$ eq.(14) N/mm$^{1.5}$	$2\bar{\beta}$	$\sqrt{GG'_c}$ eq.(25)	η	$\sqrt{GG_m}$ eq.(29)
250	100	a	4.0	2	4	3	100	1,87	14,5	1,8	19,0	2,6	55,0
								2,08	16,2		20,7	2,53	60,0
								2,14	16,6		20,3	2,4	59,4
								2,40	18,6		20,7	2,12	61,0
		b					150	1,77	13,7	1,9	20,5	2,53	71,6
								1,92	14,9		20,7	2,4	73,3
								2,37	18,4		21,4	2,12	79,3
		c					100	1,79	13,8	~ 0	21,3	2,0	43,8
								1,95	15,1		20,0	1,5	39,8
250	80	a						2,18	16,9	1,8	21,6	2,53	62,8
250	120					3		1,93	14,9		19,0	2,53	55,6
						3		1,94	15,1		19,3		55,9
					mean	6			15,0		19,2		55,8
250	100		6,0			3	100	1,85	14,3		18,3		53,3
400	100	d	4,0			3	100	1,98	17,1	~ 0	19,7	1,95	55,6
							160	1,61	15,8		24,0		49,0
150		e					90	2,63	15,8	2,2	20,2	2,53	82,4
250		f		2	2		100	1,78	13,8	1,5	18,9	2,52	51,2
								1,68	13,0		17,8		48,3
								2,11	16,3		22,4		60,6
											eq.(16)		eq.(21)
		g			4			1,89	14,6	1,0	17,9	2,48	< 139
								2,35	18,2		22,2	2,16	< 143

In series f of table 8 the nail again was probably determining.

Series g of table 8 is an end-support and fracture will occur according to the equations of the notched beams (the split off part of the beam is unloaded). However $\sqrt{GG_c}$ is a factor 20/8.1 = 2.5 higher with respect to the notched beam because there is no clear initial crack.

The equivalent critical relative crack length $\bar{\beta}$ in the middle of the beam of a connection patern at the lower part of a beam height is about 0.9 (exclusif the length of the pattern in beam direction) for an uniform spread pattern over the height αh.

For a concentrated patern (in one row in beam direction) this is about the half.

The higher value in the middle of the beam with respect to an end-support (where $\beta = 0.5$) is due to the moment carying capacity of the split off part of the beam in the middle of the beam.
Series c and d in table 8 show an early failure (β is small) and also $\sqrt{GG_m}$ is small. Comparable tests of other series show much higher values.
The boundary η_g between horizontal splitting and vertical splitting lies theoretically lower for small values of α than for higher values of α. The high values of $\sqrt{GG_m}$ in table 6 (except for the series where the joints are determining) show that η_g is below 2.37. Table 8 shows that η_g may be below 2. If it is assumed that η_g lies below 1.4 in table 7 than is:
$\sqrt{GG'_c} \approx 21$ and $\sqrt{GG_m} \approx 49$ N/mm$^{1.5}$ if both mechanisms are determining at the same time. It can be concluded that:
$\sqrt{GG_m}$ is about 49 to 53.9 and as for notches can be estimated at 50 N/mm$^{1.5}$ and:
$\sqrt{GG'_c}$ is about 20 to 21 N/mm$^{1.5}$.

A design rule could be based on an equivalent critical crack length of about $0.9 \cdot h$ (where h is the height of the beam) for connections in the middle of the span and about $0.4 \cdot h$ for connections at the end of a beam and for concentrated patterns of connections. Easier however is to base the method on an equivalent work term according to eq.(14) (because the shear-mode of cracking is strongly dominating) and to give a lower bound for all cases.
Series b of table 6 gives a mean: $\sqrt{GG_c} = 15.9$ (21 tests) and series a to e of table 8 shows a mean of 15.7 (48 tests). The coefficient of variation is about 8 %. For vertical cracking V is mainly dependent on $\sqrt{\eta}$ in eq.(30). In table 9 the values are calculated according to these design formulas:

$$\frac{V_f}{\alpha b h} = f_v \sqrt{\frac{h_v}{h}} \qquad (32)$$

and for $\eta \leq \eta_0$:

$$\frac{V_m}{\alpha b h} = f_v \sqrt{\frac{h_v}{h} \cdot \frac{\eta_0}{\eta}} \qquad (33)$$

It follows from table 9:

$$f_v \sqrt{h_v} = 34{,}1 \quad \text{and}$$

$$f_v \sqrt{h_v \eta_0} = 49{,}3 \text{ N/mm}^{1.5}$$

so that: $\eta_0 = (49{,}3/34.1)^2 = 2.1$.

Table 9. Strength of the connections according to the proposed design rules

h mm	b mm	type	d mm	num. per row	num. of rows	num. of tests	αh mm	$\frac{V}{b\alpha h}$ N/mm^2	$f_v\sqrt{h_v}$ $N/mm^{1.5}$	η	$\eta_0 f_v \sqrt{h_v}$ $\eta \leq \sim 2$ $N/mm^{1.5}$
Strength connection according to table 6 series b											
180	40	b/c	3,8	5	1	5	28	3,54	47.5	2,37	
						3		3,92	52,6		
					mean	8		3,68	49,4		
		b	"	5	1	1	47	2,85	38,2		
						3		2,54	34,1		
					mean	4		2,62	35,2		
				5	1	3	66	2,26	30,3		
							85	1,97	26,4		
							104	2,27	30,5		
Strength of the connection according to table 8 series a to e											
250	100	a	4,0	2	4	3	100	1,87	29,6	2,6	
								2.08	32,9	2,53	
								2,14	33,8	2,4	
								2,40	37,9	2,12	55,2
		b					150	1,77	28,0	2,53	
								1,92	30,4	2,4	
								2,37	37,5	2,12	54,6
		c					100	1,79	28,3	2,0	40,0
								1,95	30,8	1,5	37,8
250	80	a						2,18	34,5	2,53	
250	120					3		1,93	30,5	2,53	
						3		1,94	30,7		
250	100		6,0			3	100	1,85	29,3		
400	100	d	4,0			3	100	1,98	39,6	1,95	55,3
							160	1,61	32,2		45,0
150		e					90	2,63	32,2	2,53	
						mean			34,1		48,0

5. Proposal for design rules for the Dutch code and Eurocode

5.1 Beams with notches at the ends.

For notches at the ends of a beam applies:

$$\frac{V_{rep}}{\alpha bh} = \alpha f_v \sqrt{\frac{h_v}{h}} = \alpha \cdot 19{,}1/\sqrt{h} = \frac{2}{3} \cdot f_{v,rep} \cdot 1{,}5 \cdot 19{,}1 \cdot \alpha / (f_{v,rep} \cdot \sqrt{h}) = \frac{2}{3} \cdot f_{v,rep}\, \alpha \cdot 9{,}55/\sqrt{h}$$

with: $f_v\sqrt{h_v} = 28.5\ N/mm^{1.5}$ at failure. The representative or characteristic value is about: $(1 - 1{,}64 \cdot 0{,}2) = 0.67$ times higher or: $0{,}67 \cdot 28{,}5 = 19{,}1\ N/mm^{1.5}$. $f_{v,rep} = 3\ N/mm^2$

For small values of η is for notched beams:

$$\frac{V_{rep}}{\alpha bh} = \frac{\alpha}{\eta} f_m \cdot \sqrt{\frac{h_m}{h}} = \frac{2}{3} \cdot f_{v,rep} \cdot 1{,}5 \cdot 43{,}6 \cdot \alpha / (3 \cdot \eta \cdot \sqrt{h}) = \frac{2}{3} \cdot f_{v,rep} \cdot \alpha \cdot 9{,}55 \cdot 2{,}3/(\eta \cdot \sqrt{h})$$

with: $f_m\sqrt{h_m} = 65\ N/mm^{1.5}$ at failure or with $0{,}67 \cdot 65 = 43{,}6\ N/mm^{1.5}$ as representative (characteristic) value.
Thus the design rules can be based on:

$$V_d \le \frac{2}{3} f_{v,d}\, bh_e \cdot \frac{h_e}{h} \cdot \sqrt{\frac{90}{h}} \qquad \text{when } \eta \ge 2.3$$

and:

$$V_d \le \frac{2}{3} f_{v,d}\, bh_e \cdot \frac{h_e}{h} \cdot \sqrt{\frac{90}{h}} \cdot \frac{2.3}{\eta} \qquad \text{when } \eta < 2.3$$

where $\eta = M_d/(V_d h)$ and h is in mm.

Alternatively the last proposal of the Dutch TGB-1990 art. 11.10 or Eurocode 5 art. 5.1.7.2 can be changed to:

$$k_{jon} = 1 + (k_{kee} - 1) \cdot \left(1 - \frac{a}{3 \cdot (h - h_e)}\right)$$

where $k_{jon} = 1$ when $h_e = h$ and $a \ge 3 \cdot (h - h_e)$

$$k_{kee} = \frac{h_e}{h} \cdot \sqrt{\frac{90}{h}} \qquad \text{when } \eta \ge 2.3,$$

and:

$$k_{kee} = \frac{2.3}{\eta} \cdot \frac{h_e}{h} \cdot \sqrt{\frac{90}{h}} \qquad \text{when } \eta < 2.3$$

with: $\eta = M_d/(V_d h)$. In the Eurocode k_{jon} is denoted by k_v.

When $h > h_e \geq 0.9 \cdot h$ linear interpolation is allowed between $k_{jon} = 1$ at $h = h_e$ and the value of k_{jon} at $h_e = 0.9 \cdot h$.

5.2 Connections at the lower part of a beam

For connections at the lower part of the height of a beam applies:

$$\frac{V_{rep}}{\alpha b h} = f_v \sqrt{\frac{h_v}{h}} = \frac{2}{3} \cdot f_{v,rep} \cdot 1{,}5 \cdot 22{,}85/(3 \cdot \sqrt{h}) = \frac{2}{3} \cdot f_{v,rep} \cdot 11{,}4/\sqrt{h} = \frac{2}{3} \cdot f_{v,rep} \sqrt{\frac{130}{h}}$$

with a representative value of: $f_v \sqrt{h_v} = 0{,}67 \cdot 34{,}1 = 22{,}85 \text{ N/mm}^{1.5}$.
For $\eta \leq \eta_0$ is:

$$\frac{V_m}{\alpha b h} = f_v \sqrt{\frac{h_v}{h} \cdot \frac{\eta_0}{\eta}} = \frac{2}{3} \cdot f_{v,rep} \cdot 1{,}5 \cdot 33{,}0/(3 \cdot \sqrt{h \eta}) = \frac{2}{3} \cdot f_{v,rep} \cdot \sqrt{\frac{130}{h} \cdot \frac{2.1}{\eta}}$$

with the representative value of: $f_v \sqrt{h_v \eta_0} = 0{,}67 \cdot 49{,}3 = 33{,}03 \text{ N/mm}^{1.5}$.

For the TGB-1990 art. 13.1.4 or Eurocode art. 5.3.1 can be proposed:

$$V_d \leq \frac{2}{3} f_{v,d}\, b h_e \cdot \sqrt{\frac{130}{h}} \qquad \text{when } \eta \geq 2.1$$

and:

$$V_d \leq \frac{2}{3} f_{v,d}\, b h_e \cdot \sqrt{\frac{130}{h} \cdot \frac{2.1}{\eta}} \qquad \text{when } \eta < 2.1$$

$$V_d \leq \frac{2}{3} f_{v,d}\, b h_e \qquad \text{when } h_e \geq 0.7 \cdot h$$

where $\eta = M_d/(V_d h)$ and h is in mm.
In the Eurocode is h_e denoted by b_e and b by t.

Literature

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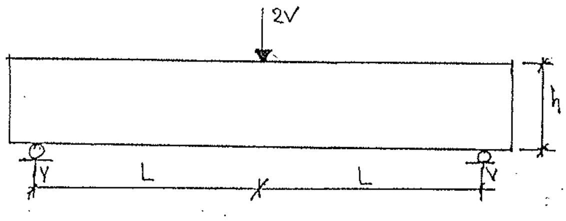

fig. 1.

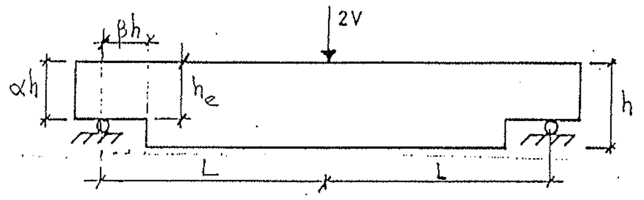

fig. 2.

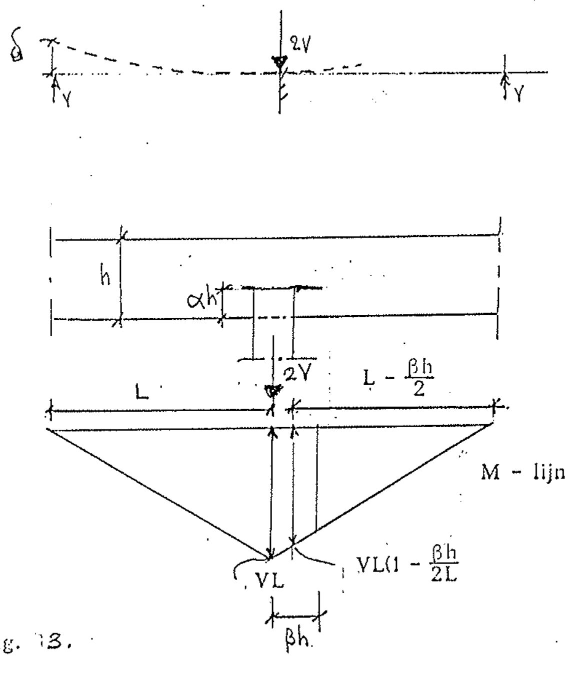

fig. 3.