Comment on: Mode II critical stress intensity factor of wood measured by the asymmetric four-point bending test of single-edge-notched specimen while considering an additional crack length, of H. Yoshihara in Holzforschung, Vol. 66, pp 989-992, 2012.

T.A.C.M. van der Put
TU Delft faculty of Civil Engineering and Geosciences, Timber structures and wood technology.
Tel: +312152851980 0613818723 e-mail: vanderp@xs4all.nl

Introduction

The results of the here discussed article of Yoshihara (2012), are unsatisfactory and need to be reconsidered. The existence of two critical stress intensities is postulated, which may differ a factor 3 to 4 from each other. A questionable push up of the lower data-curve, as given by Fig. 3 of Yoshihara (2012), may diminish this difference, but what to do with the conclusion that the critical stress intensity is effectively obtained. It is not mentioned when the high, and when the low values apply and why. Any conclusion should be based on theory and the theoretical explanation of the in Yoshihara (2012) and (2008) mentioned data, is given in van der Put (2012), because these data provide a confirmation for mode II softening like behavior. Based on this theory, correction is necessary of the interpretation and conclusion of Yoshihara (2008). This discussion is given here in the light of the achievements of wood mechanics.

Discussion of deformation and damage processes in wood.

The damage processes in wood at loading have to be discussed, because in Yoshihara (2012), (2008), a relation is assumed between the fracture strength and early deformation response, without giving a reason why and what it explains. Why is assumed that the relatively low strength of the asymmetric four-point bending (AFPB) test is related to low stress viscoelastic response of the three-point bending end-notched flexure (3ENF) test. The bend off, of the loading line of the 3ENF test is due to the start of flow of a non-linear viscoelastic process, which is not comparable with the fracture process in the end state of the AFPB test.

As known, strength and time dependent deformation of materials only can be explained by the physical and chemical processes, thus by statistical mechanics and reaction kinetics, and as shown, by limit analysis, of the in van der Put (1989) developed equilibrium theory of deformation kinetics, aspects as creep, damage, aging, annealing, nucleation (van der Put 2011b), transformations as glass transition (see van der Put (2010), rubber behavior, diffusion, etc. are explained by the same constitutive equation. The consequence is that the contradictory phenomenological models as the free volume model for glass-transition, the instability model of nucleation and the extrapolated flexible chain model, with non-existent linear viscoelastic relaxation spectra for rubber behavior and creep of materials, etc., are rejected.

Thus as consequence, time dependent behavior of wood, timber and notched wood is non-linear and the processes follow the molecular deformation kinetics equations with a correlation close to one, only determined by the measuring precision of the testing equipment, showing the molecular large number statistics. The variability occurs among different wood specimens. Every piece of wood thus is significantly different of each other and is an unique giant molecule. The parameters, as concentration, activation energy and volume, of the processes can e.g. be found by creep, relaxation, recovery, long duration and complex loading-history tests at different temperatures, loading rates and moisture contents, measuring response and hysteresis and decrease of the modulus of elasticity, etc.

Regarding fracture, creep tests of wood, at normal test conditions, may show a quick fracture process at the start of loading as follows from the high enthalpy, which is high enough for primary C-O-bond or C-C-bond rupture. The fact that this process is quick, despite the high activation energy, shows that the internal stress is high, as occurs at initial crack extension at imperfections to a stable length and direction, providing then the sites for following extension. However, this primary bond breaking process is of minor importance in the ramp loading tests of fracture mechanics. Comparable (high enthalpy) processes only dominate at a high stress level e.g. in controlled crack growth tests. Dominating during ramp loading are the viscoelastic processes. The main process (of side group readjustment by cooperative hydrogen bond breaking) is coupled to a process with a long delay time (and a very low initial flow unit density), which shows irreversible strain at an activation energy which is high enough for primary bond breaking. This second process starts at an ultimate strain of the first process and is probably due to flow of early-wood, providing stress redistribution to the latewood (because this process did not occur in cell wall tests). This also follows from the correlation of the strength of wood with the density and the correlation of the amount of latewood to the density. Regarding the response, these coupled processes show a constant activation volume parameter, independent of initial stress and temperature and thus show, outer the time-temperature equivalence, also a time stress equivalence, what means that the relaxation time decreases with increasing stress and increases with decreasing stress, what explains in the last case, the quasi permanent deformation at unloading, which only can be recovered by heating and moistening. The consequence of the decreasing relaxation time with stress increase is that the primary bond-breaking process, thus fracture, only is noticeable, within the time-scale of ramp-loading, close to the top of the loading curve. This means that the lower bend off of the loading curve of the 3ENF-test is not due to fracture, and is not correlated to fracture.

Discussion of calculation method of Yoshihara (2012), (2008)

Of the given Eq.(1) of Yoshihara (2012) the stress intensity factor K_{II} as function of a/W is also the load *P* a function of a/W :

$$K_{II} = \frac{3P}{4BW}\sqrt{\pi(a)}\, f\left(\frac{a}{W}\right) \tag{1}$$

In principle gives Eq.(5):

$$K_{II} = \frac{3P}{4BW}\sqrt{\pi(a+\Delta)}\, f\left(\frac{a+\Delta}{W}\right) \tag{5}$$

just the function value of Eq.(1) for $a+\Delta$. This does not change the function.
Also when $a+\Delta$ has to replace a, as coordinate transformation, the function and physical properties don’t change. Relatively only the ordinate axis shifts over the distance Δ along the abscissa axis. The upwards shift of the curve, (which should not occur by the chosen transformation), means that a wrong relation is given between the load *P* and the replaced increased crack length of $a+\Delta$. This can be seen as follows:
In Yoshihara (2008), the energy release rate G_{II}, according to the compliance calibration method is also given by:

$$G_t = G_{II} = \frac{P^2}{2B}\cdot\frac{\partial C}{\partial a} = \frac{P^2}{2B}\left(A_1 + 2A_2 a + 3A_3 a^2\right) \tag{2}$$

as relation of G_{II} with the measured values of *P* and *C*, while *C* uniquely connects the nominal value of a to the measurements. As shown in Yoshihara (2008), precisely the same value of G_{II} is found by Eq.(1), by determining G_{II} by the virtual crack closure method for the nominal crack length a, at the measured value of *P*. Here a is at an other manner uniquely related to G_{II} and to the measured value of P. Eq.(2) can be given as:

$$\frac{G_{II}}{P^2} = B_1 + B_2 a + B_3 a^2 \quad (2)$$

and also Eq.(1) can be written in this form:

$$\frac{G_{II}}{P^2} = C_1 a + C_2 a^2 + C_3 a^3 + C_4 a^4 + ... + C_n a^n \quad (1)$$

Equating these two equations gives the polynomial:

$$0 = -B_1 + D_1 a + D_2 a^2 + C_3 a^3 + C_4 a^4 + ... + C_n a^n$$

The n roots of this equation gives the n points where the values of G_{II} are the same of both methods, thus n points of the curve of G_{II}. If now in the same equations (1) and (2), a is replaced by $(a + \Delta)$, then, the polynomial gives precise the same n roots as for the polynomial in a, showing the ordinate axis to be shifted over the distance Δ and there is no raise of the value of G_{II} as given by Yoshihara (2008), (2012).

The dispersion of the transformed data, given in Fig. 3 of Yoshihara (2012), (2008), which depend on the chosen polynomial form confirms the error made. This, of course, is impossible by a right calculation because all chosen polynomials fit the same data.

Explanation of the apparent decrease of the fracture toughness at softening.

The only case of decrease of the stress intensity factor at testing occurs at the end of the softening state, when it is assumed that only macro-crack extension occurs of a single crack. In the 3ENF–test, there is sufficient space for the crack to extend in. Then unloading after reaching the top as softening follows the Griffith locus at the constant, maximal value of G_{II}, (as derived in van der Put (2011a), Section 3.3). For the AFPB-test, the space $(W - a)$ is too small, due to the applied too long (post-critical) initial crack-length a, and the fracture plane is soon overloaded and distant small crack propagation by crack merging may become determining because at the critical small crack density, crack merging takes less energy than macro-crack propagation (see derivation in van der Put (2011a) Section 3.6, where it also is shown that the logarithmic cracking rate of molecular deformation kinetics is followed). The situation of the post-critical long cracks of the AFPB-test is, as if the specimen was loaded over the top of the curve of Fig. 3.2 of van der Put (2011a) and then after softening, is unloaded and again reloaded for further crack extension and softening. The analysis, for softening by small crack merging is then as follows, using the equations and data of Yoshihara (2012), (2008):

Eq.(1) of Yoshihara (2012) for the top of the loading curve at $a/W = 0.7$ is:

$$K_{II,0} = \frac{3P}{4BW}\sqrt{\pi a_c} = \tau_n \sqrt{\pi a_c} \tag{1c}$$

because $f\left(a/W\right) = 1$ and the nominal Griffith shear stress is: $\tau_n = 3P/4BW$ (2c)

As follows from van der Put (2011a), eq.(3.13) is the critical crack length: $a_c = \sqrt{bl/6\pi}$ for mode I, according to fig. 3.1. For the AFPB-test is, $b = W$ and $l = c_1 W$, proportional to W, as a St. Venant distance. A similar relation applies for mode II, following from the Griffith locus derivation for compression and shear. Thus $a_c = \sqrt{Wc_1W/6\pi} = c_2 W$ (3c)

For higher values of *a* is, as applied in Yoshihara (2012),(2008):

$$K_{II} = \tau_n \sqrt{\pi a} \cdot f(a/W) \tag{4c}$$

It is shown in van der Put (2011a), Section 3.6, that at a far stage of softening, the small crack merging mechanism shows that at any moment the strength of the intact fracture area is determining for the strength. This can be interpreted that not the nominal stress but the ultimate real stress f_t in the fracture plane is determining. Thus eq.(1c) becomes:

$$f_t = K_{II,0} / (\sqrt{\pi a_c} \cdot (1 - a_c/W)) = K_{II,0} / (\sqrt{\pi c_2 W} \cdot (1 - c_2)) \tag{5c}$$

and eq.(4c): $f_t = K_{II} / (\sqrt{\pi a} \cdot (1 - a/W) \cdot f(a/W))$ (6c)

and from eq.(5c) and (6c) follows:

$$\frac{K_{II}}{\sqrt{\pi a} \cdot (1 - a/W) \cdot f(a/W)} = \frac{K_{II0}}{\sqrt{\pi c_2 W} \cdot (1 - c_2)} \text{ or:}$$

$$\frac{K_{II}}{\sqrt{a/W} \cdot (1 - a/W) \cdot f(a/W)} = \frac{K_{II0}}{\sqrt{c_2} \cdot (1 - c_2)} = c_4 \quad \text{(constant)} \tag{7c}$$

According to fig.12 of Yoshihara (2008), there is no difference (by volume effect) between the data for W = 40 mm and W = 20 mm, thus mean values of both can be regarded.

For *a/W* = 0.7, $K_{II} = 0.79$ MPa$\sqrt{m}$ thus: $c_4 = 3.15$

For *a/W* = 0.8, $K_{II} = 0.71$ MPa$\sqrt{m}$ thus: $c_4 = 3.3$

For *a/W* = 0.9, $K_{II} = 0.52$ MPa$\sqrt{m}$ thus: $c_4 = 3.28$

$f(a/W) =$ 1.0 for *a/W* = 0.7 and is 1.2 for *a/W* = 0.8 and is 1.67 for *a/W* = 0.9

Thus the mean value of c_4 is: c_4 = 3.24 Numerically this calculation is:

0.79/($\sqrt{0.7}$ (0.3)·1) = 3.15 and: 0.71/($\sqrt{0.8}$ (0.2)·1.2) = 3.31 and: 0.52/($\sqrt{0.9}$ (0.1)·1.67) = 3.28

It thus is confirmed by the data of Yoshihara (2012)(2008), that the real mean shear strength of the intact part of the fracture plane is determining and not the apparent K_{II} - value.

The value of the nominal stresses are therefore also determined by the small crack extension as follows from:

$f(a/W) = \dfrac{K_{II}}{\tau\sqrt{\pi a}} = \dfrac{K_{IIc}}{\tau_c\sqrt{\pi a_c}}$, or: $\sqrt{\pi a_c} \cdot f(a/W) = \dfrac{K_{IIc}}{\tau_c} = c_5$ (constant).

Now: $a_c = \sqrt{bl/6\pi} = \sqrt{\left(c_1 W(1-a/W)\cdot W(1-a/W)/6\pi\right)} = c_6(1-a/W)$ because $l = W(1-a/W)$ and $b = c_1 W(1-a/W)$ as St. Venant distance. Thus:

For *a/W* = 0.7, is: $\sqrt{a_c} \cdot f(a/W) = c_6\sqrt{(1-a/W)}$ $f(a/W) = c_6 \cdot \sqrt{0.3} \cdot 1.0 = 0.55 c_6$

For *a/W* = 0.8, is: $\sqrt{a_c} \cdot f(a/W) = c_6\sqrt{(1-a/W)}$ $f(a/W) = c_6 \cdot \sqrt{0.2} \cdot 1.2 = 0.54 c_6$

For *a/W* = 0.9, is: $\sqrt{a_c} \cdot f(a/W) = c_6\sqrt{(1-a/W)}$ $f(a/W) = c_6 \cdot \sqrt{0.1} \cdot 1.67 = 0.53 c_6$,

giving the explanation and control of the $f(a/W)$ - values of Yoshihara (2012), (2008).

It thus is shown that small crack merging and extension towards the macro-crack tip is always determining for fracture. ((W = 40 mm then tau-c = 4 N/mm2))

Background of the applied exact theory

The following should be accounted for the derivation of exact fracture theory:

Wood has to be regarded as a reinforced isotropic material. The existence of an isotropic matrix in wood follows not only from material analysis, but also from the high compression strength at confined dilation with the absence of failure by triaxial hydrostatic compression, (what is not the case for anisotropy, because for equal triaxial stresses, the strains then are not equal and yield remains possible). Because of this, the, in van der Put (2009) derived orthotropic critical distortional energy principle applies for initial yield, showing the start of dissipation of elastic distortional energy as also confirmed by the orthotropic finite element calculation of Gopu (1987). By this dissipation according to the incompressibility condition, the minimum energy principle is followed providing therefore the exact flow criterion

Perpendicular to the grain, isotropic behavior of the wood matrix is e.g. shown by oblique grain tests (see van der Put (1982)) which show hardening, after first orthotropic flow, leading to equal stresses, independent of orientation.

The failure criterion of clear wood and of timber (van der Put (2009), (1982); Hemmer (1985)) and failure criterion of a single notch (van der Put (2011a)), is the same, showing that the small-crack extension towards the macro-crack tip is the cause of macro-crack extension. This is confirmed by the fact that the stress intensity factor is the same independent on the macro-form and dimensions of the notch. It also is confirmed by molecular deformation kinetics, showing the same processes in clear- and notched wood. Also the solutions given in this article of the strength behavior of long post-critical crack length is totally based on small crack behavior. The small-crack merging mechanism also explained precisely the mode I softening curves of Boström (1992). The failure criterion shows no coupling term between the normal stresses at "flow", and thus shows no dowel action of the reinforcements and there only is a direct interaction of the reinforcement with the matrix and the matrix stresses determine the stresses of the reinforcements.

Exact solutions are always possible according to limit analysis, which is based on an elastic-full plastic (full fractured) approach. This means that in stress space, the flow criterion is a single curve and for "plastic" dissipation, the stress tensor should be along the concave curve what means that the extremum variational principle applies for "flow" and thus the virtual work equations apply and thus the theorems of limit analysis and thus lower and upper bound solutions exist for any allowable equilibrium system, following as solution of the Airy-Stress function.

Fracture of wood is a common boundary value problem of the strength at the crack boundary. This is derived in van der Put (2011a) Chapter 2, and it appears that, for any load combination, fracture occurs by reaching the uniaxial tensile strength at the crack boundary. This uniaxial stress is a measure of the cohesion strength and leads to the mixed mode Wu-equation:

$$K_I / K_{Ic} + \left(K_{II} / KI_{IIc}\right)^2 = 1$$

as exact solution, as well of the isotropic Airy stress function of the matrix, as of the orthotropic Airy stress function of the total stresses and the Wu-equation thus is no longer an arbitrary empirical equation, but is the highest lower bound solution which is equal to the real solution. The by the theory predicted oblique crack extension and interference with approaching small cracks, causes the by Wu (1967) measured displacement of the crack boundary and the skipping over fibers. Because at the crack boundary, or at the elastic-plastic

boundary, the elastic solution as well as the ultimate strength solution applies, it is necessary to solve the elastic solution of the isotropic matrix stresses and then calculate the stresses in the reinforcement in proportion to the modulus of elasticity of matrix and reinforcement. This way of calculation is important for wood because of the direct interaction of the reinforcement and matrix, causing failure of the matrix, to be determining for the start of failure. This follows e.g. for Balsa wood, which is highly orthotropic, but is light, thus has a low reinforcement content and thus failure shows the isotropic ratio of $K_{IIc} / K_{Ic} \approx 2$ of the isotropic matrix material. Also for stronger wood which is failing by oblique crack extension, it is possible that the start of crack extension may show the isotropic oblique angle, showing the matrix to be determining for initial failure, etc.
It is therefore a requirement for an exact orthotropic solution, applicable to wood, to also satisfy the equilibrium condition of the matrix-stresses. The mathematical flat crack solution of Sih, can be modified to represent such failure criterion at the crack-tip boundary $r = r_0$, for uniaxial loading. The mixed mode criterion is however a linear relation between the stresses, which therefore does not apply for wood because the parabolic Wu-equation is the exact solution which is measured.

Conclusion

- It is shown that push up of the K_{IIc} - curve of Fig. 3 of Yoshihara (2012) is not possible. The replacement of a / W by $(a + \Delta) / W$ is a transformation along the a / W -axis.
- An exact explanation is given of the apparent decrease of the (single crack) stress intensity factor. This appears to be caused by a small crack merging mechanism, which takes much less energy than a single macro-crack extension. The data of Yoshihara (2008), (2012) follow the theory precisely. Probably this mechanism also determines always the strength of timber.
- The properties of wood should be accounted for a right fracture mechanics theory. It therefore is necessary to regard:
--that wood behaves as a reinforced material, and the solutions of the isotropic Airy stress function of the matrix as well as the orthotropic Airy stress function are needed.
-- that reaction kinetics and the general applicable failure criterion show that, small-crack behavior is always determining for fracture.
-- that for overloaded fracture planes of long post-critical initial cracks, the strength of the intact part of the fracture plane is always determining due to the determining small crack merging mechanism which explains the factor 2.5 to 3.9 too low stress intensity factors of

Yoshihara (2008), (2012). This behavior is predicted to be identical to softening behavior for common initial crack lengths of the same specimens (see van der Put (2011a)).

References

Boström L. (1992), Method for determination of the softening behavior of wood etc. Thesis, Report TVBM-1012, Lund, Sweden,

Gopu, Vijaya K. A. (1987) Validity of Distortion-Energy-Based Strength Criterion for Timber Members, *J. Struc. Eng.* 113, No. 12 pp. 2475-2487.

Hemmer K. (1985) Versagensarten des Holzes der Weisstanne unter mehrassige Beanspruchung, Dissertation, University of Karlsruhe, Karlsruhe.

Put T.A.C.M. van der (1982) A general failure criterion for wood, Proceed. 15th CIB-IUFRO Timber Eng. Group Meeting, Boras, (Google title + name author)

Put T.A.C.M. van der (1989) Deformation and damage processes in wood, Delft University Press, The Netherlands.

Put T.A.C.M. van der (2007) A new fracture mechanics theory of orthotropic materials like wood. Eng Fract Mech 74:771–781.

Put T.A.C.M. van der (2009) A continuum failure criterion applicable to wood, J Wood Sci 55:315–322.

Put T.A.C.M. van der (2010): “Theoretical derivation of the WLF(= glass transition)- and annealing equations”, Journal of Non-Crystalline Solids 356 (2010) 394–399 .

Put T.A.C.M. van der (2011a) A new fracture mechanics theory of wood, Nova Science Publishers, New York 2011.

Put T.A.C.M. van der (2011b): “A new theory of nucleation”, in Phase Transitions, A Multinational J., 84:11-12, 999-1014.

Put T.A.C.M. van der (2012) Discussion of: “Mode II fracture mechanics properties of wood measured by the asymmetric four-point bending test using a single-edge-notched specimen of H Yoshihara in Eng. Frac. Mech. 75 pp 4727-4739” in Eng. Frac. Mech. 90 pp 172-179

Valentin G.H., Bostrom L., Ranta-Maunus A., Gowda S.(ed.), (1991) Application of fracture mechanics to timber structures, RILEM state-of-the-art report, Espo.

Wu EM (1967) Application of fracture mechanics to anisotropic plates. J Appl Mech 34:967–974.

Yoshihara H. (2008) Mode II fracture mechanics properties of wood measured by the asymmetric four-point bending test using a single-edge-notched specimen, Eng. Frac. Mech. 75 pp.4727-4739.

Yoshihara, H. (2012) Mode II critical stress intensity factor of wood measured by the asymmetric four-point bending test of single-edge-notched specimen while considering an additional crack length, Holzforschung, 66, pp 989-992.