Critics on the thesis and Heron article of Raven et al

Discussion of the article in Heron 53, No. 1, (2008) by: W.J. Raven, J. Blaauwendraad and J.N.J.A. Vamberský: "Elastic compressive-flexural-torsional buckling in structural members" and of their thesis: "Nieuwe blik op kip en knik":
by:
T.A.C.M. van der Put, TU Delft CiTG Timber Engineering

1 - Extension of the comment on the Heron 53 No 1 article

Comment on the Heron 53-1 article was given in Heron 53 (2008) No. 3. Due to the given space limitations, most problems only could be mentioned and subsequent remarks and remarks on the thesis are given here with extensions in chapter 2.
This discussion uses the same notation, literature references and equations numbers as the Heron 53-1 article and the Heron 53-3 comment on the article. For new references upper case letters are used.
The stability approach of the article is based on the equations of the Dutch Timber Code [12], being extensions of the Chen and Atsuta equations for eccentric loading along the beam axis. These TGB-equations are simplified in an unique way to retain the determining coupling term in all circumstances and the solution satisfies the failure criterion. This last is not followed in the article and a not general applicable integrated form of the equations is used, applied to a fictive, (Euler-type) central normal and lateral loading case, only in the stiff direction, thus without primary loading for lateral buckling. In the main direction this leads to the first of Eqs.(17) and (18a), of a free supported, lateral rigidly supported beam, loaded by centric loads without a second order effect for bending, while also the second order effect of compression is left out of the lateral buckling equations (18b) (to obtain uncoupling of the equations) leading to a fictive case, without primary moments $M_{x1} = M_{z1} = 0$ and zero eccentricities $\varphi_0 = e_y = 0$. Thus only an impossible, unsafe case is treated, based on wrong differential equations (18a) and wrong solutions by wrong splitting of variables and wrong superposition, n-values, failure criterion, etc. etc. <u>The theory is nowhere applied rightly in thesis and article as is shown here (giving the proof)</u>. Application thus certainly leads to building damage. This is corrected here as a necessary completion of the thesis.

Following the comment of Heron 53-3 on the Heron 53-1 article, the most severe mistakes to be discussed with increasing importance, are:
- 1. The elastic stability approach is claimed to be followed in the article although the TGB-failure criterion of severe flow is used named "unity check" or "ultimate state" in the thesis. Elastic critical instability does not exist for structural building elements. It even is not thinkable for lateral buckling (called flexural torsional buckling by the authors) where the large deformation analysis shows that the deformation remains increasing at increasing loading in the elastic buckled stage. As shown by buckling

and lateral buckling tests (Stevin-laboratory, missing in the thesis), the ultimate state occurs. Therefore, the applied linear bending stress diagram has to be adapted to the occurring nonlinear elastic-full plastic diagram by profile factors in accordance with the so derived failure criterion. Also the matching apparent E-modulus then has to be determined from this ultimate state. (See [B] and paragraph D below).

- 2. The analysis of article and thesis is wrongly based on not general applicable 2nd degree differential equations in stead of the right 4th degree differential equations of the problem. From the 6 equilibrium equations, the forces are eliminated so that three differential equations of the 4th degree of the moments remain. The applied twice integrated 2nd degree equations with zero integration constants only apply for free supported beams and cannot at the same time apply for fixed supports and other boundary conditions as is done. Thus a right application for real loading cases and boundary conditions is prevented in thesis and article.

- 3. The serviceability condition is wrongly applied. According to international agreement, the subjective and rather personal free serviceability limit has to be based on linear elasticity and not on a stability calculation, as done in the thesis and article, because this leads in fact to a second ultimate state criterion by which countries may keep out foreign constructors and manufacturers of the Union (by rejecting their products being not based on their requirements of this second failure criterion).

- 4. The severe nonsense written in 8.2, of the thesis or in 5 of the article about normative stresses, shows that it is not known that the failure criterion [12] is a straight line approach, Eq.(f), of the failure curve following from the ultimate elastic-plastic stress-distribution in the critical cross section due the loading F, M_y, M_z, that - with exception of the Timber Code, which only applies for simple cases – has to be approximated by 2 straight lines, giving the right failure criterion for every situation. The applied straight line cut off of the Code, called unity check by the authors is not a superposition of stresses, as given in Fig. 13, and is not, contradictory at the same time, a totally different sum of weighted or "normative" stresses (Eq.(40)). Fig. 13 thus is wrong because it does not show the ultimate elastic-plastic stress state that determines the expression (Eq.(f)) of the failure criterion. One of the terms of this "unity check"-equation is for compression and the other for bending, thus also authors reply in Heron 53 – 3, that the compression stress has to be added to the bending stress, is not right. This of course is not possible in the plastic zone. (See also discussion in paragraph D and E)

- 5. Superposition is not allowed for stability, also not in the elastic stage. Thus, Eqs.(39a/b) etc. are wrong. The analysis should be based on the total load of the different determining critical load combinations. This means that the examples given in the article are useless (never determining) and the proposed iteration method is too lengthy and practical impossible in praxis by the complicated load- and moment distributions along the beam axes and the only rational method is to sum up the first expanded Fourier term of the primary loads or primary moments of the determining loading combinations. This means that in the article only one loading case of a first expanded should have been regarded making all future calculations and iterative procedures superfluous. Because this solution, based on the first expanded, (or mean of the moment surface of the middle part of the beam) already is known, even in the necessary format of the interaction equation of the ultimate state (TGB-method), it is a major mistake to force people to invent the wheel again for every single calculation with all sorts of illicit means as superposition, linking of stiffness, splitting of variables, illegal disregard of initial eccentricities, plasticity and failure etc., etc.

- 6. Because of the elastic and plastic non-linearity, the series and parallel linking of n-values (stiffness) is not valid, even not in the treated most simple, not existing Eulerian loading case of the thesis and article where all eccentricities and M_z etc. are zero because then (see paragraph B):

$$\frac{1}{n}=\frac{F}{F_{ez}}+\left(\frac{M_y}{M_k}\right)^2=\frac{F}{F_{ez}}+\left(\frac{M_y}{M_{k0}}\right)^2\cdot\frac{1}{(1-F/F_{ey})(1-F/F_T)} \quad (34)$$

This is not a simple linking of n-values because F_{ey} and F_T are not negligible (e.g. for thin walled profiles with nearly equal stiffness in both main directions). This also follows solely from M_c (a modification of M_k, [B]) for a distributed load:

$$M_c=\frac{\pi}{L}\sqrt{\frac{EI_zGI_t\left(1+\frac{\pi^2}{L^2}\frac{EI_w}{GI_t}\right)\left(1-\frac{F}{F_t}\right)\left(1-\frac{F}{F_{ey}}\right)\left(1-\frac{F}{F_{ez}}\right)}{1-\frac{I_z}{I_y}}}x$$

$$x[\sqrt{\left(\frac{e_{m0}}{2}\right)^2\frac{F_{ez}}{GI_t}\frac{\left(1-F/F_{ey}\right)\left(1-F/F_{ez}\right)}{\left(1-F/F_t\right)}+1}\;-$$

$$-\sqrt{\left(\frac{e_{m0}}{2}\right)^2\frac{F_{ez}}{GI_t}\frac{\left(1-F/F_{ey}\right)\left(1-F/F_{ez}\right)}{\left(1-F/F_t\right)}}\;]$$

The discussion of “n” for real, existing, loading cases is given in paragraph A and B.

- 7. The interaction equation between failure by buckling and by lateral buckling is the critical stability equation for all materials. It is the goal of the calculation and demanded justification of the safety for all materials. Only for timber this is not an empirical equation but is derived from the theory as is the necessary basis of a calculation. This theoretical method and equation is lacking in the article what is a serious not allowed decline. (See discussion in paragraph D and E below).

- 8. The chosen splitting of variables in advance in the thesis and article is not allowed and is a capital blunder. The splitting according to Eq.(9) of the article:

$$v=v_0+v_1+v_2 \quad (9)$$

of the variable “v” into an initial value v_0; a first order displacement by the external load v_1 and a second order displacement v_2, is impossible because the components: v_1, v_2 of “v”, follow the second order equation Eq.(a) and v_1 then can not satisfy the first order equation, Eq.(b), as well. v_1 in Eq.(b) is a function of M_{z1} only, while v_1 in Eq.(a) is a function of $v_2, w_1, w_2, \varphi_1, \varphi_2, F, M_{x1}, M_{y1}, M_{z1}$. Stating the v_1's to be the same is stating the differential equations (a) and (b) to be the same (= to be dependent). <u>To obtain the second order displacements, the solution of Eq.(b) has to be subtracted from the solution of Eq.(a)</u>.

Further, the splitting of variables cannot be based on the treated fictive loading case, Eq.(17) as done, because for $M_{x1} = M_{z1} = 0$, only a trivial solution of Eq.(b): $v_1 = \varphi_1 = 0$ is possible. Non-trivial solutions follow from the real occurring cases of [12], given in [14] as Eq.(4.07). Then the splitting in the Heron 53-1 article of:

$$w = w_1 + w_2;\ v = v_1 + v_2;\ \varphi = \varphi_1 + \varphi_2$$

(when initial values are left out for shortness of illustration), leads to:

$$\begin{aligned} &EI_y(w_1 + w_2)'' + Fw - M_{x1}v' - M_{z1}\varphi + M_{y1} = 0 \\ &EI_z(v_1 + v_2)'' + Fv + M_{x1}w' + M_{y1}\varphi + M_{z1} = 0 \\ &-GI_t(\varphi_1 + \varphi_2)' - M_{z1}w' + M_{y1}v' + M_{x1} + M_{x2} = 0 \end{aligned} \qquad \text{(a)}$$

The displacements with index 1 is expected to also follow the linearized equations:

$$\begin{aligned} &EI_y(w_1)'' + M_{y1} = 0 \\ &EI_z(v_1)'' + M_{z1} = 0 \\ &-GI_t(\varphi_1)' + M_{x1} = 0 \end{aligned} \qquad \text{(b)}$$

As mentioned this is impossible because w_1, v_1 and φ_1 can not satisfy Eq.(a) and Eq.(b) at the same time. Substitution of Eq.(b) into Eq.(a) for a solution, as done in the article, gives Eq.(4.08) of [14] and Eq.(18a) (for small values of M_{x1} and M_{z1}):

$$\begin{aligned} &EI_y(w_2)'' + F(w_1 + w_2) - M_{x1}(v_1 + v_2)' - M_{z1}(\varphi_1 + \varphi_2) = 0 \\ &EI_z(v_2)'' + F(v_1 + v_2) + M_{x1}(w_1 + w_2)' + M_{y1}(\varphi_1 + \varphi_2) = 0 \\ &-GI_t(+\varphi_2)' - M_{z1}(w_1 + w_2)' + M_{y1}(v_1 + v_2)' + M_{x2} = 0 \end{aligned} \qquad \text{(c)}$$

This however is only a partial elimination of index 1 values. However the authors solved the total displacement of these partial eliminated, meaningless Eq.(c) in a wrong way. This is discussed below at Eq.(e) and at paragraph A.

A necessary further elimination of index 1 deformations to solve the index 2 deformations of Eq.(c) gives for the first of Eq.(c):

$$\begin{aligned} EI_y(w_2)'''' + F(w_2)'' - M_{x1}(v_2)''' - M_{z1}(\varphi_2)'' = F(M_{y1}/EI_y) - M_{x1}(M_{z1})'/EI_z + \\ -M_{z1}(M_{x1})'/GI_t) \end{aligned} \qquad \text{(d)}$$

Because the author's reply is not convinced of Eq.(d), the derivation in the smallest possible steps is given below at paragraph C. Eq.(d) shows that the primary moments are still in the second order equation, now in a wrong way. The index 1 displacements have no meaning and have to be omitted and index 2 displacements like v_2 should be read as $v_2 = v - v_0$, where v is the total displacement following as solution of Eq.(a). The second order moments thus don't follow from the wrong Eq.(c), that for $M_{x1} = M_{z1} = 0$ is equal to Eq.(18a) e.g.: $EI_y(w - w_0)'' + Fw = 0$ with the solution $M_{y2} = nFw_0 / (n-1)$, but follow from the subtraction of the solution of the linearized equation Eq.(b) from the result of the solution of the non-linear equation, Eq.(a). Thus $M_{y2} = n(M_{y1} + Fw_0) / (n-1) - M_{y1} = (M_{y1} + nFw_0) / (n-1)$. Thus $M_{y1} / (n-1)$ is not accounted in M_{y2} in thesis and article. This also leads to a wrong definition of n = n*.

- 9. The definition by n* in Eq.(12), (13) has no meaning because it is an identity, replacing v_2 by v/n*. Thus n* is a superfluous shortcut for $v / v_2 = (v_0 + v_1 + v_2) / v_2$. It thus has nothing to do with the multiplying factor "n" because that factor cannot be stated in advance but follows as the solution of the total displacement "v" from the differential equations Eq.(a). Inserting the shortcuts Eq.(12a,b): $(n^*)_y = n_y = w / w_2 = w / (w - w_0)$ and $(n^*)_z = n_z = v / v_2 = v / (v - v_0)$ with $M_{x1} = M_{z1} = 0$, in Eq.(c) gives:

$$
\begin{aligned}
&EI_y w'' / n_y + Fw = 0 \\
&EI_z v'' / n_z + Fv + M_y \varphi = 0 \qquad \text{(e)} \\
&-GI_t \varphi' + M_y v' = 0
\end{aligned}
$$

As following blunder, these equations, Eq.(e), being Eq.(6.08) of the thesis, are solved for constant values of n_y and n_z what is impossible because e.g. $n_z = v / v_2 = v / (v - v_0)$ can not be constant because v_0, the initial value of "v" can not be proportional to "v". This so called n*-method, the new approach of the article and thesis, thus is not right. The real meaning of n* is derived below in paragraph A.

In article and thesis the mistake is made to apply the iterative numerical solution on the fictive loading case: $M_{x1} = M_{z1} = 0$ giving only the trivial solution of Eq.(b): $v_1 = \varphi_1 = 0$ and to regard this as a real solution. Lucky, this mistake of a mistake leads to the right values $v_1 = \varphi_1 = 0$ for Eq.(a) because the index 1 displacement have no meaning in Eq.(a). This means that splitting of variables is not applied on the coupled equations of Eq.(a), giving therefore the right solution. The authors reply thus is not right. <u>The finite element method does not proof the splitting of variables to be right, but to be wrong, because it confirms the common analysis with no split variables.</u> The splitting only is applied on the uncoupled equation for "w" and w_1 is solved from Eq.(b) leading only to a wrong failure condition Eq.(40):

$$\frac{F}{F_u} + \frac{M_{y1}}{M_{uy}} + \frac{M_{z2}}{M_{uz}} = 1 \qquad (40)$$

with a not multiplied first order moment M_y. This has to be:

$$\frac{F}{F_u}+\frac{M_y+Fw_0}{M_{uy}(1-F/F_{Ey})}+\frac{M_{z2}}{M_{uz}}=1 \tag{f}$$

This wrong failure criterion Eq.(40), applied in the calculation examples of article and thesis, also is inserted in the finite element solution of the thesis what explains the same result of both identical calculations despite the error.

The second term of Eq.(f) may also dominate in the failure criterion when the stiffness of both main directions are not far apart (in columns), leading to severe unsafe errors when Eq.(40) is applied. The authors' reply that this is only 1% is of course absurd when F approaches both F_{Ey} and F_{Ez}.

- 10. The lack of knowledge what the failure criterion is leads to other unpredictable errors as for instance in 6.2, the calculation example. It is terrible that it is possible to extend the failure criterion with a fourth term with M_{zfl}:

$$\frac{F}{F_u}+\frac{M_y}{M_{uy}}+\frac{M_z}{M_{uz}}+\frac{M_{zfl}}{M_{uzfl}}=1 \tag{g}$$

This moment should be added to M_y or to M_z, depending of the direction of M_{zfl}. Because M_{zfl} is the self-equilibrium bimoment, $M_{zfl} = 0$, causing a zero bending moment on the section (see pg 274 of [C]).

The failure criterion Eq.(f) is no summation of stresses and also no summation of meaningless relative stresses but is the result of plastic flow of nearly the whole cross section due to the external loading F, M_y and M_z independent of an internal stress redistributions by an internal equilibrium system. It is incredible that, after the extended discussion and delivering of 15 pages on the failure criterion in 2001, (See par. G) and after the extended discussion with respect of the thesis and the extended discussion of the reviewer of the Heron article, it still is not known by the author's reply what the failure criterion is, and still is not known that it is not the result of an incomprehensible kind of summation of stresses.

By applying simplified Code TGB-equations, the approach of the thesis is meant for the same simple cases. However, the Code is safe in all circumstances what is not the case for the approach of the article because of the mentioned mistakes; the possibility of a wrong applied failure criterion also not accounting for the influence of a reduced shear-capacity due to the plastic zone of the cross section with the absence of safety against torsional instability (e.g. when $I_z \approx I_y$); the negligence of initial eccentricities and initial loading M_{x1}, M_{z1} and of F_{ey} and F_T in Eq.(34), Eq.(40) and Eq.(f), etc. what surely will be disastrous when determining.

2. - Additional comment correction and explanation

A. - The inadmissibility of the so called n*-method

As mentioned above the given definition of the n-values shows that they can not be

constant in Eq.(e) and an other meaning should be found. This meaning of n* can be derived as follows. .
Eq.(e) are solved in the thesis by giving a zero value to the nominator determinant. This means that n* = n_z should be regarded as such a reduction factor of the stiffness EI, that the general solution becomes determining at the given loading in stead of the normally determining particular solution.
The coupled equations (4.07) or (5.01) of the thesis are:

$$EI_z(v-v_0)''+Fv+M_y\varphi=0 \qquad (5.01)$$
$$-GI_t\varphi'+M_yv'+M_x=0$$

The homogenous parts for the general solution thus are:

$$EI_zv''+Fv+M_y\varphi=0 \qquad (5.01b)$$
$$-GI_t\varphi'+M_yv'=0$$

The general solution of Eq.(501b) follows from the substitution of:
$v=Ae^{\lambda x}$ en $\varphi=Be^{\lambda x}$,
and by the zero value of the numerator determinant giving the value λ of the characteristic equation according to:

$$\lambda=\pm i\frac{\pi}{L}\sqrt{\frac{F}{F_E}+\frac{(M_y)^2}{(M_{cr})^2}} \qquad =i\lambda'$$

with $F_E=\pi^2EI_y/L^2$ and $M_{cr}^2=\pi^2EI_yGI_t/L^2$.
The total solution of v, including the particular solution then is:

$$v=A\sin(\lambda'x)+B\cos(\lambda'x)+\frac{v_0}{1-\frac{F}{F_E}-\frac{M_y}{F_EGI_t}}$$

In this case with a sinus form of v_0, $v_0=A\sin(\pi x/L)$, only the particular solution applies because $\lambda'<\pi/L$, because F and M_y remain below their Euler values by the presence of the initial displacements and because the strength is reached earlier.
When v_0 approaches zero, the general solution (with B = 0) approaches the particular solution with $\lambda'=\pi/L$ and is:

$$\frac{F}{F_E}+\frac{(M_y)^2}{(M_{cr})^2}=1$$

This can be seen as follows. From the particular solution follows that:

$$\frac{v-v_0}{v}=\frac{F}{F_E}+\frac{M_y^2}{F_EGI_t}=\frac{1}{n_p}$$

so that the general solution can be given like:

$$v=A\sin\left(\frac{\pi\cdot x}{L}\sqrt{\frac{v-v_0}{v}}\right)=A\sin\left(\frac{\pi\cdot x}{L}\sqrt{\frac{1}{n_p}}\right)$$

For $v_0\to 0$, this approaches to the particular solution with $\lambda=\pi/L$. The same can be done for the solution of the thesis equations Eq.(5.01c). In this equation EI_y is reduced to EI_y / n_z giving:

$$\lambda = \pm i \frac{\pi\sqrt{n_z}}{L}\sqrt{\frac{F}{F_E}+\frac{(M_y)^2}{(M_{cr})^2}} = \pm i \frac{\pi}{L}\sqrt{\frac{n_z}{n_p}}$$

The general solution giving n_z, becomes determining when $n_z = n_p$, or:

$$\frac{1}{n_z} = \frac{1}{n_p} = \frac{F}{F_E}+\frac{(M_y)^2}{(M_{cr})^2}$$

The virtual stiffness reduction factor $n_z = n^*$ thus is equal to "n" of the second order multiplying factor n/(n-1) on v_0. However, this only is right when there is no primary loading and when $\varphi_0 = 0$. In the same way, when φ_0 is not zero, only the right value $n_z = n_p = n^*$ applies when $v_0 = 0$. Further, equating the nominator determinant to zero, only gives one equation with one unknown. Thus for a coupled system, n_z for the displacement w, only can be obtained when $v_0 = \varphi_0 = 0$.
For a real non Eulerian case, when initial eccentricities are not zero, e.g. when v_0 and φ_0 are not zero, "n" only can calculated from the determining particular solution which is e.g. in this case:

$$\frac{1}{n} = \frac{v - v_0}{v} = \frac{M_y\varphi_0 + Fv_0 + \dfrac{M_y^2 v_0}{GI_t}}{F_E v_0 + M_y\varphi_0}$$

Clearly the n*-method does not apply for real loading cases but only for the fictive case of the partial uncoupled equations with $\varphi_0 = 0$, without eccentricities and primary loading, thus $M_{x1} = M_{z1} = 0$, thus the only case discussed in thesis and article. In the iterative calculation of thesis and article, "n" rightly is determined from the particular solution although it is superfluous to determine this virtual uniaxial value of "n" because no conclusions follow from it. The only criterion is the from "v" calculated moment that, together with other loading should satisfy the failure criterion.

B. - The uselessness of the Eulerian uniaxial multiplication factor n/(n-1)
The uniaxial multiplication factor n/(n-1) does not exist in a simple explicit form in the always occurring triaxial loading case (by the initial eccentricities and heterogeneities) because this factor then is a complicated expression of the loading, thus showing no bifurcation and the failure behaviour is comparable with the common bending test causing the writing n/(n-1) senseless and superfluous.
For the most simple biaxial loading case of the twofold eccentric compressed straight Euler column there is no explicit solution of the characteristic equation, giving loading dependent Euler values. For the symmetric cross section this follows in this case from the characteristic equation on pg. 163 of [C]:

$$(EI_z\lambda^2 + F)(EI_y\lambda^2 + F)(EI_\omega\lambda^2 - GK_t - \bar{K}) - (Fe_z)^2(EI_y\lambda^2 + F) + \\ -(Fe_y)^2(EI_z\lambda^2 + F) = 0$$

Based on the first expanded terms of the Fourier expansion, most of the 12 integration constants can be taken to be zero and the sinus-function remains as solution. This approaches to the particular solution with: $\lambda = i\pi / L$, when v_0 approaches zero. With the equivalent torsional rigidity:

$GI_{ve} = \pi^2 EI_\omega / L^2 + GK_t + \bar{K}$

the characteristic equation becomes, with: $F_{Ez} = \pi^2 EI_z / L^2$, etc.:

$$-(F_{Ez} - F)(F_{Ey} - F)GI_{ve} + (M_z)^2(F_{Ey} - F) + (M_y)^2(F_{Ez} - F) = 0$$

If one of the eccentricities is zero, e.g. when $M_z = 0$, the equation is equal to the thesis equation (for n=1). For double eccentricity this is:

$$\frac{(M_z)^2}{GI_{ve}(F_{Ez} - F)} + \frac{(M_y)^2}{GI_{ve}(F_{Ey} - F)} = 1 \quad \text{or:} \quad \frac{(M_z)^2}{(M_{cz})^2} + \frac{(M_y)^2}{(M_{cy})^2} = 1$$

where $M_{cz} = \sqrt{GI_{ve}F_{Ez}(1 - F / F_{Ez})}$ and $M_{cy} = \sqrt{GI_{ve}F_{Ey}(1 - F / F_{Ey})}$

are loading dependent by F, because splitting as separate term in F is not possible. These expressions of M_c, dependent on F, show that the parallel and series linking of 1/n according to the equations (5.15), (5.16), (5.92), (5.9), (8.15), etc. of the thesis (and Eq.(34) to (39) of the article) are not right. Only for the simplest possible, fictive Euler case of thesis and article is for high beams approximately:

$$\frac{1}{n} = \left(\frac{M_y}{M_c}\right)^2 + \frac{F}{F_E},$$

from what it is wrongly concluded that always in general applies:

$$\frac{1}{n} = \frac{1}{n_M} + \frac{1}{n_F} \quad \text{(called, parallel linking).}$$

The real occurring equivalent value of "n" follows from the moment Eq.(11') on pg. 7 of [B]. having the form:

$$M_{z,F} = \frac{\sum c_i M_i}{N} = \sum \frac{M_i}{1 - 1 / n_i}$$

whereby N is the expression of the nominator. From this follows that:

$$1 - \frac{1}{n_i} = \frac{N}{c_i} \quad \text{or} \quad \frac{1}{n_i} = 1 - \frac{N}{c_i}$$

The nominator term is:

$$N = \left(1 - \frac{F}{Fez}\right)\left(1 - \frac{e_v M_y}{GI_v}\right) - \left(\frac{M_y}{M_k}\right)^2 = 1 - \frac{F}{Fez} - \left(1 - \frac{F}{Fez}\right) \cdot \left(\frac{e_q M_y + e_p M_z}{GI_v}\right) - \left(\frac{M_y}{M_k}\right)^2.$$

In this equation is:

$F_{ez} = \pi^2 EI_z / L^2$;

$GI_v = \pi^2 EI_\omega / L^2 + GI_t + K = GI_t(1 + \pi^2 EI_\omega / \{GI_t L^2\} - F(I_y + I_z) / \{GI_t A\}) =$

$\quad = GI_t(1 + \pi^2 EI_\omega / \{GI_t L^2\})(1 - F / F_t)$

$GI_m = GI_v(1 - F / F_{ey}) / (1 - EI_z / EI_y)$ and $M_k = \sqrt{F_{ez} \cdot GI_m}$.

“e” and “s” are eccentricities of vertical and horizontal the lateral loading “q” and “p” respectively.
The denominator terms $c_i M_i$ of $M_{z,F}$ are:

$$Fv_0\left(1-\frac{e_q M_y + e_p M_z}{GI_v}\right)+F_{ez}v_0\left(\frac{M_y}{M_k}\right)^2+\varphi_0 M_y + M_z\left(1+\frac{M_y(s_p-e_q)}{GI_v}\right)+$$
$$+\frac{M_y^2\cdot s_q}{GI_v}-\frac{M_z^2\cdot s_p}{GI_v}$$

where the denominator moments M_i, are the primary moments by p and q.
The most simple contribution to $M_{z,F}$ of the term $\varphi_0 M_y$ is:
$n_\varphi M_y \varphi_0/(n_\varphi-1)$, (Thus $c_i=1$) and

$$\frac{1}{n_\varphi}=\frac{F}{F_{ez}}+\left(1-\frac{F}{F_{ez}}\right)\cdot\left(\frac{e_q M_y+e_p M_z}{GI_v}\right)+\left(\frac{M_y}{M_k}\right)^2$$

This relation also applies for the last 2 denominator terms and for the second denominator term.
For the equivalent multiplication factor of $F v_0$, thus: $n_F F v_0/(n_F-1)$ is:

$$\frac{1}{n_F}=1-\frac{1-\frac{F}{F_{ez}}-\left(1-\frac{F}{F_{ez}}\right)\cdot\left(\frac{e_q M_y+e_p M_z}{GI_v}\right)-\left(\frac{M_y}{M_k}\right)^2}{1-\frac{e_q M_y+e_p M_z}{GI_v}}$$

From this all follows that <u>there is no simple parallel and series linking of 1/n</u> in this slightly more general loading case than the trivial one of the thesis and article as is to be expected from elastic non-linearity. The Eulerian n/(n -.1) expression for each term is thus immensely more complicated than the direct solution and the method is not applicable even not for simple loading cases.
It thus is necessary for real cases to apply the simplification by means of an interaction curve as is applied in the TGB and is applied for all other building materials. In the thesis, only the trivial case with zero $e_p, e_q, s_p, s_q, \varphi_0$ and M_z is treated and the only remaining denominator terms then are:

$$Fv_0 \text{ and } F_{ez}v_0\left(\frac{M_y}{M_k}\right)^2$$

and the value of 1/n becomes as simplest possible (Eulerian) case:

$$\frac{1}{n}=\frac{F}{F_{ez}}+\left(\frac{M_y}{M_k}\right)^2=\frac{F}{F_{ez}}+\left(\frac{M_y}{M_{k0}}\right)^2\frac{1}{(1-F/F_{ey})(1-F/F_T)}.$$

showing still no possible series of parallel linking as discussed in paragraph 6.

C. - Derivation of Eq.(d)

Because the displacements can not satisfy both independent equations Eq.(a) and

Eq.(b), there is no unique solution possible. Thus the solution depends on the way followed at elimination of w_1, v_1 and φ_1 from Eq.(a). To eliminate these variables from Eq.(c), this equation has to be differentiated twice, thus:

$EI_y(w_2)''''+F(w_2)''+F(w_1)''-(M_{x1}(v_1+v_2)')''-(M_{z1}(\varphi_1+\varphi_2))''=0$, giving for the term: $(M_{x1}(v_1+v_2)')''=((M_{x1})'(v_1+v_2)'+M_{x1}(v_1+v_2)'')'=$

$=(M_{x1})''(v_1+v_2)'+2(M_{x1})'(v_1+v_2)''+M_{x1}(v_1+v_2)'''=$

$=2(M_{x1})'(v_1+v_2)''+M_{x1}(v_1+v_2)'''$, because $(M_{x1})''=0$ by the constant eccentricity.

The last term is:

$(M_{z1}(\varphi_1+\varphi_2))''=((M_{z1})'(\varphi_1+\varphi_2)+M_{z1}(\varphi_1+\varphi_2)')'=$

$(M_{z1})''(\varphi_1+\varphi_2)+2(M_{z1})'(\varphi_1+\varphi_2)'+M_{z1}(\varphi_1+\varphi_2)''=.$

$=2(M_{z1})'(\varphi_1+\varphi_2)'+M_{z1}(\varphi_1+\varphi_2)''$, because $(M_{z1})''=q_y=0$ of no horizontal loading.

Substitution from Eq.(b): $(w_1)''=-M_{y1}/EI_y$; $(v_1)''=-M_{z1}/EI_z$; $(\varphi_1)'=-M_{x1}/GI_t$

gives: $EI_y(w_2)''''+F(w_2)''-FM_{y1}/EI_y-2(M_{x1})'(v_2)''+2(M_{x1})'M_{z1}/EI_z+$

$-M_{x1}(v_2)'''+M_{x1}(M_{z1})'/EI_z-2(M_{z1})'(\varphi_2)'+2(M_{z1})'M_{x1}/GI_t+$

$-M_{z1}(\varphi_2)''+M_{z1}(M_{x1})'/GI_t=0$. or:

$$EI_y(w_2)''''+F(w_2)''-M_{x1}(v_2)'''-M_{z1}(\varphi_2)''-2(M_{x1})'(v_2)''-2(M_{z1})'(\varphi_2)'=$$

$$=F(M_{y1}/EI_y)-2(M_{x1})'(M_{z1})/EI_z-M_{z1}(M_{x1})'/GI_t-2(M_{z1})'(M_{x1})/GI_t \qquad \text{(d-1)}$$

It is evident that this is far away from the real second order moment. To come closer hereto, terms with moment gradients should be zero by regarding pure bending. Pure bending dominates in praxis (between 2 lateral supports of large beams). For pure bending: $(M_{x1})'=0$ and for only end moments: $(M_{z1})'=0$ and Eq.(d-1) becomes in general by weak moment gradients:

$$EI_y(w_2)''''+F(w_2)''-M_{x1}(v_2)'''-M_{z1}(\varphi_2)''\approx$$

$$=F(M_{y1}/EI_y)-2(M_{x1})'(M_{z1})/EI_z-M_{z1}(M_{x1})'/GI_t \approx F(M_{y1}/EI_y) \qquad \text{(d-2)}$$

Finally for the non existent loading case of thesis and article, thus for $M_{x1}=M_{z1}\equiv 0$ and $\varphi_0=\varphi_1\equiv 0$, Eq.(d-2) becomes:

$$EI_y(w_2)''''+F(w_2)''=F(M_{y1}/EI_y) \qquad \text{(d-3)}$$

Because M_{y1} is constant, also $(w_1)''$ is constant and $(w_1)''$ is now no variable of Eq.(a) and subtraction of a constant is possible before integration. Thus: the first of Eq.(a): $EI_y(w)''+Fw=-M_{y1}$ is, after subtraction of the constants $EI_y(w_1)''=-M_{y1}$, equal to $EI_y(w-w_1)''+F(w)=0$, being the first of Eq.(c). The solution however, is not possible in $w-w_1=w_2$, but only in the total displacement "w" and the second order w_2 stills follows (as generally) from the subtraction of w_1 from w.

The authors reply is wrong. There was no full substitution of the index 1 variables into Eq.(a) and the equation then was not solved as done here in the right way.

D.- The calculation of the critical moment of lateral buckling

The lateral buckling calculation is a second order strength calculation for timber structural elements. (This is a demand of Eurocode 1 and TGB-algemeen). In the determining case of the in the TGB applied failure criterion there is plastic "flow" of a large part of the cross section, much giving a higher strength with respect to the (elastic) first flow state. The influence of the extension of the plastic area along the beam length can be calculated (see ref. [A]), with aid of a simple relation derived in the appendix of [B] and therewith the equivalent E-modulus is known. The, from this following linear calculation method of the strength thus is a calculation code. The linear bending strength is calculated from the elastic-plastic diagram (see appendix of [B]). That is why profile factors are necessary to adapt the linear bending strength to the real elastic-plastic stress distribution in the cross section of a beam. All measurements of the combined bending-compression strength on pg. 33 of [B] can be explained in this way accounting for the volume effect for tension. This also applies for the bending strength of the full size stability tests of the Stevin-laboratory (available as students report for a real thesis). The in the Raven thesis applied TGB-criterion is a criterion for far extended plasticity as applies e.g. for indoors applied not treated wood due to the always occurring mechanosorptive effect. The other extreme case, when for a bad (not applicable) quality the bending tensile strength is determining and linear behaviour occurs up to fracture, the failure criterion is totally different (see pg. 33 of [B]). It can be seen that then the bending strength even increases at the application of compression. (For wood, this of course is absent in the thesis, but the same can be seen in the empirical interaction curve of concrete on pg. 233 of the thesis).

In the thesis, wrongly the Euler stiffness factor, e.g. $\sigma_{m,cr}$ at pg. 146, is regarded as the lateral buckling strength. The lateral buckling strength however follows from the solution of M_y, from the equation of the failure criterion (a third degree equation, see [B], Eq.(15')) giving:

$$\frac{F}{f_c A}+\frac{M_y+Fw_0}{f_b W_y(1-F/F_{Ey})}+\frac{v_0\left(F+F_{Ez}\dfrac{M_y^2}{M_c^2}\right)+s_m F_{Ez}\dfrac{M_y^2}{M_c^2}}{f_b W_z\left(1-\dfrac{F}{F_{Ez}}-\dfrac{M_y^2}{M_c^2}\right)}=1$$

This relation is given here, just as in the thesis, for $M_{z0}=0$, while s_m and M_c are non-linear functions in F (see [B], pg. 5 and Eq.(14)).

The determination of this lateral buckling strength or critical buckling moment is the goal of the calculation. This is absent in the dissertation so that therefore reference has to be made to [A] and [B]. The failure criterion:

$$\frac{F}{F_u}+\frac{M_{y,total}}{M_{uy}}+\frac{M_{z,total}}{M_{uz}}=1$$

is an approximation of the curve of combined bending-compression strength ([B], pg.33) for indoors applied, unprotected, high quality timber. This is not a linear superposition of stresses as stated in the thesis and is not at the same time a summation of stresses divided by their uniaxial strengths, as given in Fig. 8.1:

$\sigma_c + \sigma_{my} + \sigma_z = \sigma_{tot}$

as superposition. Although the failure criterion has nothing to do with superposition, real superposition of loading is not possible for the non-linear buckling case. Eq.(5.03) thus is not right. It shows the necessity to apply the first Fourier-expanded or to apply the TGB mean loading over the mid half part of the beam. (This last is the case because the work is proportional to $(\sin(\pi x / L))^2$, having the form of a step-function over the mid half of the beam). Thus all bending stresses around the y-axis have to be summed and then divided by uniaxial strength as done here above by: $M_y + Fw_0$.

E. - The buckling – lateral buckling interaction equation

The thesis claims to give one calculation method for all materials, but the contrary is true. The same method for lateral buckling is already applied as long as theory exists by the buckling-lateral buckling interaction failure criterion, called interaction equation, giving a relation in terms of pure buckling and pure lateral buckling. This relation is determined empirically for other materials, what is not right by the demand to base the calculation on the theory. For wood, this interaction relation is derived by elimination of the initial displacements v_0, w_0, φ_0 from the 3 failure criteria of compression (pure buckling), bending (pure lateral buckling) and combined bending-compression (lateral buckling) giving one relation in terms of pure buckling and pure lateral buckling. Only the pure buckling and pure lateral buckling cases then are functions of the initial displacements and the bending strength automatically approaches the Euler values for very slender beams and the bending strength for stocky beams and the pure buckling strength remains below the compression strength. This calculation of the critical values and of the interaction curve is lacking in the dissertation.

It is destructive that the authors of the dissertation don't know what the interaction relation for buckling and lateral buckling is, stating in publications that it is a wizard relation with no physical meaning and teaches this to their students.

F. - The proposed TGB-correction

Because the stability calculation of the CIB-Eurocode draft (and EC 5 draft), was not sufficient, it was decided by the TGB-committee, 22 years ago, to follow a general method, simplified as much as possible. A derivation was necessary because no general method existed then, accounting for stocky beams and all profile forms and warping influences as pure torsional buckling (e.g. for partial instability of short beams between the lateral supports of long beams and for thin walled profiles with $EI_y = EI_z$) with all possible eccentrically loadings and all initial displacements whereby the failure criterion was satisfied. The, from this derived interaction relation between buckling and lateral buckling, shows that stresses are always below the bending strength and compression strength. This was tested by consultants and designers around 1990. Curious is that now suddenly in the thesis is stated that the pure lateral buckling of the TGB is above the bending strength, even despite of the condition $k_{ins} = 1$ for slenderness below that of the bending test specimen. Thus below 160, because L/h = 18 and h/b =2.56 and $\lambda_z = \sqrt{12} \cdot L / b = \sqrt{12} \cdot 46 = 160$. This means that for lower slenderness according to Fig.B8.3 and B8.4 of the thesis, not the right strengths are inserted what, by the applied wrong failure criterion (by the w_1-variable splitting) led to too low strengths. The precise adaptation of the iterative calculation and the Ansys calculation in figures B8.3 etc. only is possible by the insertion of the same wrong failure criterion in the finite element method. These lower strength values only partially explain the lower values with respect to the TGB-values. The figures B8.3 to B8.6 show an interpretation mistake of too high pure TGB - lateral buckling strengths (on the horizontal axis) where the kinking point is displaced proportionally. The error thus is solely in k_{ins}. When this lateral buckling strength is reduced to the same Ansys-strength value, the kink moves proportionally along the line through the zero point and the kink point and remains below the Ansys line as to be expected by the in 2 directions regarded and eliminated initial displacements and by the safe neglecting of terms of the theoretical equation. (Only the kink of the left Fig. B8.5 is wrong by a calculation error). In the pre-publication of thesis and article in the Houtconstructeur, this lateral buckling strength was found to be right, but then the kinking point was found to be totally wrong (by a calculation error). Now it thus is found to be the other way around. Because the error of the thesis of the lateral buckling strength is not found for slender beams, the error is caused by an error in k_{ins} for low slenderness and thus by an error in the adaptation formula to the bending strength of the bending test (prescribed in 1986 by a marking line besides the text in the CIB-Eurocode draft) or by the interpretation of the loading. This loading is based in the TGB on the mean moment of the middle part of the beam. In the TGB, different from the Eurocode, the occurring stress is not compared with the first order stress so that not the critical values should be enlarged with the theoretical moment distribution factor (table 1 of [B]). It is clear that this is done in the figures of the thesis, giving too high values by a factor of about 1.3 for a point loading and a factor 1.1 for a distributed loading. This has to be corrected and the performed TGB-correction should be rectified.

G. - History

The thesis of WJ Raven was published before in 2001 and was rejected as method by the TGB-Timber Committee. An extended correction of 30 pages by this Committee with comments, including 15 pages on the failure criterion and wood mechanics, was addressed to Raven. However, his rejected uncorrected paper was published in the Houtconstructeur, (now as “Uitgave 01-3”), while the TGB-correction was banned from publication. Comparable extended comments were later made on the Thesis and still later on the article for Heron. Derivations, as given here in chapter 2, were proposed, in order to bring corrections as further developments of the first working hypotheses of the thesis. Astonishing, the article was accepted by Heron without any correction of the severe errors (in mathematics and wood mechanics), against the judgment of the review, (showing Heron to be a friends club and not a scientific journal). The reviewer was the expert on this matter having written the TGB Timber Code and derived 20 years ago, the by Raven wrongly applied special stability equations with the conditions of applicability and has done the experimental research of damage increase on full scale timber beams with the for the theory necessary perfect boundary conditions. Astonishing again, also the comment on the thesis and on the article can not be published in media of the Timber designers Union (the “Vereniging van Houtconstucteurs”) and the Board of the Union even did not react on the regular appeal-procedure of a member against this censorship. This means that the board of the “Vereniging van Houtconstucteurs” is fully responsible for the damage caused by the application of this method of the thesis. It further means that a warning is necessary that Dutch timber designers probably can not be trusted by the severe lack of information and of education (by the authors).

Literature

[A] - Comment in name of the Code TGB-committee, on the publication of W.J. Raven: “Stabiliteitscontrole van houten staven” and calculation proposal for the TGB. Later also published in the Journal Houtconstructeur 8-27, dec. 2001.
This comment of about 30 pg. is a report in 2001 for the TGB-committee with the followed correspondence by T.A.C.M. v.d. Put
[B] - TU-Report 25.4-92-07/A/HC-8: Stability of beams, background of the TGB-rules may 1992, T.A.C.M. van der Put.
This covers about reference [22] of the thesis and [12] of the Heron article 2008-1.
[C] - Theory of beam-Columns Vol. 2: space behaviour and design, 1977 W.F. Chen and T. Atsuta

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[12] - Van der Put, T.C.A.M., Stability of beams (background of Dutch code TGB) Chap. KH 10 in PAO Continuing Education Course "Structures in timber", Delft 1992.
[14] - Raven, W.J., Nieuwe blik op kip en knik, Stabiliteit en sterkte van staven, doctoral thesis, Delft 2006,. http://repository.tudelft.nl/file/204111/173135